PDA
Public Domain Algorithms
Version 0.5
Programmer’s Manual
Abstract

The PDA library provides numerical algorithm code that can be used to replace NAG in open source software.
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**B Adding further routines**

**C Changes**

**C.1 Changes from version 0.4 to 0.5**

**C.2 Changes from version 0.3 to 0.4**
1 Introduction

This is a preliminary version of the PDA library. PDA is intended to replace the NAG library in Starlink application code. A number of people are working on this project, and as their contributions become available the library will slowly approach version 1.0.

The library is not intended as a service to Starlink users or as a full NAG replacement, but some users may find routines in this library useful.

The library is coded in Fortran and has a Fortran 77 binding. Mostly, the interface is for double precision, the Fourier transform part provides for both double and single precision, and the routines from DIERCKX exist only for single precision.

The hints for migration from NAG to this library are incomplete. Application programmers are encouraged to check these hints and to report on their experience in converting applications from NAG to this library.

NAG is a registered trade mark of The Numerical Algorithms Group. In this document the term “NAG library” refers to the NAG Fortran Library and the NAG Graphics Library. Terms like “NAG format”, “NAG array”, “weights used for NAG” refer not to any product of The Numerical Algorithms Group, but to the data format that Starlink applications use in order to call the NAG library. Similarly, terms like “NAG code” refer to Starlink application code that calls the NAG library, not to the NAG library itself.

2 Packages from which the routines were obtained

GAMS – the Guide to Available Mathematical Software on the World Wide Web – was used to identify suitable user-callable routines, and the source code was retrieved by following ‘fullsource’ anchors on the Web. GAMS is an index to individual subroutines from a variety of packages, which in turn are located at several different repositories, such as NETLIB.

One major building block of the library is FFTPACK found in NETLIB (at Oak Ridge National Laboratory and AT&T Bell Laboratories). It has been stripped down to just the bits needed to serve Starlink applications, i.e. routines to take the forward and backward FFT of a complex or purely real sequence of values. Some extra subroutines have also been written to perform N-dimensional FFTs, and to convert arrays of Fourier coefficients between NAG and FFTPACK formats. Also, a double precision version has been created.

Most routines are from the SLATEC library – a large Public Domain library – and retrieved from the CAMSUN repository at the National Institute of Standards and Technology (NIST).

NMS is another Public Domain library at the TIBER repository (also at the National Institute of Standards and Technology). Some routines are from MINPACK and retrieved from NETLIB. This too is a Public Domain package.

OPT (from NETLIB) is a less homogeneous package. PDA uses two modules:

- Simann (PDA_SA) is a Simulated Annealing algorithm written by William L. Goffe at the University of Southern Mississippi. It can be used freely for research, commercial distribution is not allowed. Goffe highly recommends to run the algorithm with the test problem with different values for the parameters, before one tries it on a “real” problem. ‘The experience you gain will be quite helpful.’
• Subplex (PDA_SUBPLX) is written by Tow Rowan at the University of Texas as Austin. It can be used without restrictions.

The gridded 2-D polynomial surface fitting routines PDA_DB2INK and PDA_DB2VAL originated from Ronald Boisvert of the US National Bureau of Standards. They were part of CMLIB from CAMSUN.

The ungridded 2-D surface fitting routines PDA_IDBVIP and PDA_IDSFFT were originally module TOMS526 of the TOMS library at NETLIB.

Some routines are from the DIERCKX library. The author Paul Dierckx calls it FITPACK, but in GAMS they have another library of the same name. FITPACK should be considered as public domain software and consequently it can be used freely for research purposes under existing conditions of appropriate referencing. It cannot be used for commercial purposes without the author’s written consent.

Although the code of the PDA library comes from other sources in the Public Domain, problems with the routines should in the first instance be taken up with Starlink and not the original authors. If there are bugs in PDA, then the first assumption must be that they were introduced during the integration of the Public Domain code into the PDA library, and that the original authors are not to blame.

### 3 Routine naming

Library routines have lower-case file names, capitalised file names are used for auxiliary source code like test programs.

Routines keep the same name as they have in the package they are retrieved from, except that ‘pda_’ is prepended. Name conflicts are usually due to similar versions of routines in different source packages. In these cases the latest or best versions are adopted.

### 4 I/O and error handling

The terminal I/O and error handling have been reviewed. Since Starlink applications will often run under some environment which results in them being detached from standard Fortran units, these areas have to be addressed and made compliant with Starlink methods of message and error handling.

A few of the FFT routines (not the actual FFTPACK routines) have a status argument. It is returned as zero if all went well, and as one if an error occurred.

The DIERCKX/NETLIB routines appear not to make terminal output. Error handling takes the form of returning to the caller with a diagnostic argument set to the appropriate value. These routines therefore have not been changed.

The MINPACK/NETLIB and OPT/NETLIB routines seem to behave similarly. Again no change was made to these routines. It may, however, be necessary for the calling routine to choose argument values such that printing of messages is suppressed.
There was a STOP statement in PDA_RMARIN. This would be executed if either of the two seeds for initialisation of the random number generator were out of range. These seeds are passed by the user to the routine PDA_SA. Instead of printing a message and stopping the program, PDA_SA will now return STATUS equal to 1. STATUS should be given as zero.

The NMS/TIBER routine PDA_UNCMND was changed so that it does not call PDA_XERROR any more. It would have used this to issue warning and error messages. It also returns a diagnostic argument with the same information, so that now it is up to the caller to check and interpret that value. There remain two STOP statements in the PDA_UNCMND algorithm, in routines PDA_D1FCND and PDA_D2FCND. These are dummy routines and never actually called.

The surface fitting routines PDA_DB2INK and PDA_DB2VAL call a routine PDA_DBVAL2. Originally, PDA_DBVAL2 called XERROR when a problem was encountered. These calls have been commented out. The routine now sets variable IFAIL appropriately instead. On exit from PDA_DB2INK and PDA_DB2VAL, STATUS is set to 1 if the IFAIL value indicated that a serious fault had occurred. STATUS must have the value 0 when the routines are called.

The surface fitting routines PDA_IDBVIP and PDA_IDSFFT, together with their support routines, have been modified to ensure that STATUS is set to 1 if the internal error status variable ISTAT indicates that a problem was encountered.

The major work in adapting the library to Starlink error reporting is to do with the SLATEC/- CAMSUN routines. These routines also form the major part of the library. The SLATEC error handling procedure would be to call PDA_XERMSG. Depending on the severity level passed to this routine and depending on the error report control flag in a global variable, PDA_XERMSG might or might not print messages, and it might or might not execute a Fortran STOP statement.

PDA_XERMSG has been re-written. It now has an additional integer argument in which it returns the a status value of 1. It will also call EMS_REP with a message constructed from the library name, routine name, and message text passed to PDA_XERMSG. PDA_XERMSG will not execute any STOP statement, but always return control to the caller. Since level-2 errors were always considered fatal, routines calling PDA_XERMSG may need to be changed to cope with regaining control after level-2 errors.

Routines calling PDA_XERMSG had to be changed to accommodate the extra returned argument. They also have to pass that status argument back up to their caller. And they must actually return control to their caller before running into exceptions that might crash the program.

5 Machine dependencies

SLATEC and NMS encapsulate machine dependencies in the same set of two routines. PDA_I1MACH contains machine-specific integer constants, and PDA_D1MACH contains machine-specific double precision constants. Use of these is rare, PDA_I1MACH is mostly asked for the Fortran unit number for printing messages. For single precision constants there would be a third routine PDA_R1MACH, which is so far not in the library.

The versions of PDA_D/I1MACH from SLATEC are superior to those from NMS. They have later revision dates, and they include cases for both DEC Alpha IEEE and Sun. In the library two source files exist, pda_d/i1mach.f_sun4_Solaris and pda_d/i1mach.f_alpha.OSF1. In usual
Starlink manner the ‘makefile’ uses the SYSTEM environment variable to pick the right source when building the library.

MINPACK uses the routine PDA_DPMPAR, which provides a subset of the information available from PDA_D1MACH. PDA_D1MACH is the preferred routine, but both exist in the library. PDA_DPMPAR was changed to call PDA_D1MACH.

It is not known how OPT and DIERCKX depend on machine specifics.

### 6 Test programs

pda_test.f is a program that calls all user-callable routines in the library. The command

```bash
% f77 pda_test.f libpda.a -L/star/lib 'ems_link'
```

should succeed. The test consists of the successful linking, and an error message indicates that the library is incomplete or has inconsistent module names. The compiled program cannot be executed.

Erfplot.f can be compiled and linked against PDA and PGPLOT. It produced the title graph.

```bash
% f77 Erftest.f libpda.a -L/star/lib 'pgplot_link'
% ./a.out
```

For the FFT routines Ffttest.f can be compiled and linked. It has to be linked with PDA and NAG. Ffttest.f convolves two test arrays by multiplying their Fourier transforms. This is done using NAG routines, and then using FFTPACK routines, and the differences between the results (together with timings) are displayed. Timings are averaged over 2000 convolutions. The commands

```bash
% f77 Ffttest.f -L/star/lib -lnag 'pda_link'
% ./a.out > temp
% diff temp Ffttest.out
```

should indicate whether the FFT routines work properly. The output will not be exactly as in the distributed file, since it depends on the platform, CPU load, etc.

There are various other test routines included in the PDA distribution:

- Covartest.f - Tests PDA_NSCOR, PDA_V11 and PDA_COVMAT.
- E02cbfe.f - Tests PDA_CHE2R.
- Lintest.f - Tests PDA_LSQR.
- Nonlin2test.f - Tests PDA_DQED.
- Nonlintest.f - Tests PDA_DNLS1E and PDA_DENORM.
- Normtest.f - Tests PDA_PPND16.
- Randtest.f - Tests the random number generators.
- Simann.f - Tests PDA_SA.
- Sorttest.f - Tests the sorting routines.
- Subplex.f - Tests PDA_SUBPLX.
- Sums1.f - Tests PDA_SUMSL.
- Sf2dtest.f - Tests PDA_DB2INK and PDA_DB2VAL.
- Sf2dtest2.f - Tests PDA_IDBVIP and PDA_IDSFFT.

These test programs can be compiled and linked as follows:

```
% f77 -o <prog> <prog>.f -L/star/lib `pda_link`
```

Some of these test programs write results to standard output. For such programs the PDA distribution includes a file with name `<prog>.out` containing a set of “standard” results with which your own results can be compared.

### 7 Linking with the library

At a Starlink node the library is available as an archive of object modules. Since it is intended primarily for Starlink application packages, the link command used is most probably ‘alink’. In that case link as follows:

```
% alink a-task.o -L/star/lib `pda_link_adam`
```

The library can equally well be used by ordinary programs:

```
% f77 program.f -L/star/lib `pda_link`
```

The `pda_link` and `pda_link_adam` scripts results in your program being linked with the Starlink Error Message Service (EMS). When an error report is to be made, the library will call EMS_SETC and EMS_REP, and you have to link your program against a version of these routines.

If you do not want to link against EMS, then you can provide your own replacements for the two EMS routines. Use the following code:

```
* File name might be mymsg.f

SUBROUTINE EMS_SETC( MESSG )
CHARACTER * ( * ) MESSG
WRITE( *, * ) MESSG
```
Finally, if your site is not a Starlink site, you can customise the library as such to make EMS obsolete. For this you have to replace the error handling routine PDA_XERMSG in the library. The new code should be:

```
* File name would be pda_xermsg.f

SUBROUTINE PDA_XERMSG( LIBRAR, SUBROU, MESSG, NERR, LEVEL, STATUS )
  CHARACTER * ( * ) LIBRAR, SUBROU, MESSG
  INTEGER NERR, LEVEL
  INTEGER STATUS
  WRITE( *, * ) LIBRAR // '/' // SUBROU // ': ' // MESSG
  STATUS = 1
END
```

You might also modify the link script pda_link so that it does not refer to ems_link any more and does not execute any awk command:

```
# N.B. the previous line should be blank.
echo -lpda
```

## 8 Fast Fourier transform (FFT)

The routines for fast Fourier transform (and their origin) are:

- **PDA_RFFTI, PDA_DRFFTI** (FFTPACK/NETLIB)
  Initialize PDA_(D)RFFTF and PDA_(D)RFFTB.

- **PDA_RFFTF, PDA_DRFFTF** (FFTPACK/NETLIB)
  Forward transform of a real periodic sequence.

- **PDA_RFFTB, PDA_DRFFTB** (FFTPACK/NETLIB)
  Backward transform of a real coefficient array.
• **PDA_CFFTI, PDA_DCFFTI** (FFTPACK/NETLIB)
  Initialize PDA_(D)CFFTF and PDA_(D)CFFTB.

• **PDA_CFFTF, PDA_DCFFTF** (FFTPACK/NETLIB)
  Forward transform of a complex periodic sequence.

• **PDA_CFFTB, PDA_DCFFTB** (FFTPACK/NETLIB)
  Unnormalised inverse of PDA_(D)CFFTF.

• **PDA_R2NAG, PDA_DR2NAG**
  Convert real FFTPACK FT to NAG format.

• **PDA_NAG2R, PDA_DNAG2R**
  Convert real NAG FT to FFTPACK format.

• **PDA_C2NAG, PDA_DC2NAG**
  Convert complex FFTPACK FT to NAG format.

• **PDA_NAG2C, PDA_DNAG2C**
  Convert complex NAG FT to FFTPACK format.

• **PDA_NFFTF, PDA_DNFFTF**
  Forward transform of a complex, N-dimensional data array.

• **PDA_NFFTB, PDA_DNFFTB**
  Backward transform of a complex, N-dimensional coefficient array.

### 8.1 Differences between NAG and FFTPACK

- FFTPACK expects and returns data in a different format to NAG.

- FFTPACK includes initialisation routines (PDA_RFFTI and PDA_CFFTI) which should be called prior to the other routines, but which don’t need to be called again until the size of the data array changes. There are no equivalent NAG initialisation routines. The main NAG FFT routines (e.g. C06FAF, etc.) do this initialisation each time they are called, irrespective of the array size.

- FFTPACK has separate forward and backward transform routines, whereas NAG only has forward routines (backward transforms are performed by using complex conjugation with the forward transform). This means that there is probably no need to supply equivalents to the complex conjugation NAG routines (a trivial operation anyway).

- FFTPACK can accept arrays of any length, whereas NAG puts some restrictions on the array length (no prime factor larger than 19 allowed in the array size, and the total number of prime factors must be less than 21).

- FFTPACK routines require a differently sized work space array.

- FFTPACK has no error checking.
8.2 Data formats for FFTPACK and NAG

This section describes the differences between the way NAG and FFTPACK store arrays of Fourier coefficients. In the following, the Fourier transform of an array of \( N \) data values is represented by a sequence of \( N \) complex values \([A_0+i*B_0], [A_1+i*B_1], ..., [A(N-1)+i*B(N-1)]\).

The differences are basically in the organisation of the Fourier coefficients within the returned array, and also in the normalisation. The normalisation of the FFTPACK values is such that doing a forward transform followed by a backward transform will result in the original array values being multiplied by a factor of \( N \).

Routines to do conversions between FFTPACK and NAG formats have been added to the library.

8.2.1 Fourier transforms of sequences of purely real values

The relevant NAG routines are C06FAF and C06FBF (the “Hermitian” routines), and the FFTPACK routines are PDA_DRFFTI, PDA_DRFFTF and PDA_DRFFTB. These routines take advantage of the symmetries present in the Fourier transform of a purely real sequence. Only half of the real (A) and imaginary (B) terms need to be calculated and stored because the other halves are just the same. This means that only half the space is required to store the Fourier transform (i.e. \( N \) elements rather than \( 2*N \)), and it takes roughly half the time to evaluate. The disadvantage is that the resulting Fourier transform array can be rather more difficult to use than if all the real and imaginary parts are stored explicitly. There are routines PDA_DR2NAG and PDA_DNAG2R to do in-situ conversions between NAG and FFTPACK format. Note, each of these routines divides the supplied values by \( \sqrt{N} \), so successive calls to PDA_DR2NAG and PDA_DNAG2R do not leave the original data unaffected (they are divided by \( N \)). This is done to cancel the effect of successive calls of PDA_DRFFTF and PDA_DRFFTB which multiplies the original data by \( N \).

The real and imaginary coefficients produced by PDA_DRFFTF are numerically larger than the corresponding C06FAF coefficients by a factor of \( \sqrt{N} \), and are ordered differently in the returned arrays. Both routines return \( A_0 \) (i.e. the DC level in the array) in element 1. PDA_DRFFTF then has corresponding real and imaginary terms in adjacent elements, whereas C06FAF has all the real terms together, followed by all the imaginary terms (in reverse order):

\[
PDA_{DRFFTF}: \quad A_0, \quad A_1, B_1, \quad A_2, B_2, \quad A_3, B_3, \quad \ldots
\]
\[
C06FAF: \quad A_0, \quad A_1, A_2, A_3, \quad \ldots, \quad B_3, B_2, B_1
\]

The zeroth imaginary term \( B_0 \) always has the value zero and so is not stored in the array. Care has to be taken about the parity of the array size. If it is even, then there is one more real term than there are imaginary terms (excluding \( A_0 \)), i.e. if \( N = 10 \), then the coefficients are stored as follows:

\[
PDA_{DRFFTF}: \quad A_0, A_1, B_1, A_2, B_2, A_3, B_3, A_4, B_4, A_5
\]
\[
C06FAF: \quad A_0, A_1, A_2, A_3, A_4, A_5, B_4, B_3, B_2, B_1
\]

If \( N = 9 \), then the coefficients are stored as follows:

\[
PDA_{DRFFTF}: \quad A_0, A_1, B_1, A_2, B_2, A_3, B_3, A_4, B_4
\]
\[
C06FAF: \quad A_0, A_1, A_2, A_3, A_4, B_4, B_3, B_2, B_1
\]
8.2.2 Fourier transforms of sequences of complex values

The relevant NAG routine is C06FCF and the FFTPACK routines are PDA_DCFFTI, PDA_DCFFTF and PDA_DCFFTB. These routines take the Fourier transform of a general complex sequence of N values (i.e. 2*N real values), also returning the Fourier transform in a sequence of N complex values. FFTPACK and NAG differ in that FFTPACK stores the real and imaginary parts of each complex value in adjacent elements of the array, whereas NAG has two separate arrays, one for the real terms and one for the imaginary terms. There is also a difference in the normalisation of the routines in that the real and imaginary Fourier coefficients produced by PDA_DRFFTF are numerically larger than the corresponding C06FAF coefficients by a factor of SQRT(N). There are subroutines [PDA_DNAG2C] and [PDA_DC2NAG] to convert between NAG and FFTPACK format. Successive calls to PDA_DC2NAG and PDA_DNAG2C will result in the original data being divided by N. This is done to cancel the multiplication by N which occurs when successive calls to PDA_DCFFTF and PDA_DCFFTB are made.

8.3 Replacing calls to C06FAF

C06FAF is the NAG routine for finding the FFT of a one-dimensional sequence of real data values. The routine performs a forward transform, storing the FFT as a “Hermitian” sequence in which only half of the real and imaginary terms are kept. The inverse transform is obtained by calling C06FBF, which finds the FFT of a one-dimensional Hermitian sequence.

The following steps are involved in replacing C06FAF calls with equivalent FFTPACK calls:

- **Increase the size of the work array**
  The work array passed to the FFT routine needs to be increased in size from N elements (for NAG) to 2*N+15 (for FFTPACK).

- **Replace call to C06FAF with [PDA_DRFFTF]**
  Replace the call
  
  ```
  DOUBLE PRECISION X(N), WORK(N)
  CALL C06FAF( X, N, WORK, IFAIL )
  ```

  with

  ```
  DOUBLE PRECISION X(N), WORK(2*N+15)
  CALL PDA_DRFFTF( N, X, WORK )
  ```

- **Add calls to [PDA_DRFFTI] if necessary**
  The work array supplied to PDA_DRFFTF needs initialising before calling PDA_DRFFTF. This is done by calling PDA_DRFFTI:

  ```
  DOUBLE PRECISION WORK(2*N+15)
  CALL PDA_DRFFTI( N, WORK )
  ```

  There is no need to re-initialise WORK if the value of N has not changed since the previous call to PDA_DRFFTI (and if the contents of the work array have not been altered). No harm will occur (except for significant slowing down of execution) if the WORK array is unnecessarily re-initialised, but it is a good idea to include some logic to prevent this.
- **Convert output (frequency domain) data to NAG format**
  Compared to the Fourier coefficients created by NAG, those created by FFTPACK are stored in a different order in the output array and are normalised differently. You can either modify your application to use the FFTPACK format throughout, or call the *PDA_DR2NAG* routine to convert the FFTPACK results into NAG format.

  ```fortran
  DOUBLE PRECISION X(N)
  CALL PDA_DR2NAG( N, X )
  ```

  where X is the output from PDA_DRFFTF. On return, X holds a NAG-style Hermitian sequence.

### 8.4 Replacing calls to C06FBF

C06FBF is the NAG routine for finding the FFT of a one-dimensional Hermitian sequence such as created by C06FAF. The routine performs a forward transform, but it is usually used to perform an inverse transform by preceding it with a call to C06GBF to form the complex conjugates of the input (frequency domain) data.

The following steps are involved in replacing C06FBF calls with equivalent FFTPACK calls:

- **Convert input (frequency domain) data to FFTPACK format**
  Compared to the Hermitian sequences created by NAG, those created by FFTPACK are stored in a different order and are normalised differently. You can either modify your application to use the FFTPACK format throughout, or call the *PDA_DNAG2R* routine to convert the supplied NAG format data into the equivalent FFTPACK format data:

  ```fortran
  DOUBLE PRECISION X(N)
  CALL PDA_DNAG2R( N, X )
  ```

  where X is the supplied NAG-style data. On return, X holds the FFTPACK-style data, ready for use by PDA_DRFFTB. If this call is made, the values returned by PDA_DRFFTB will have the same normalisation as the original data supplied to PDA_DRFFTF.

- **Increase the size of the work array**
  The work array passed to the FFT routine needs to be increased in size from N elements (for NAG) to 2*N+15 (for FFTPACK).

- **Replace call to C06FBF and C06GBF with PDA_DRFFTB**
  Replace the two calls:

  ```fortran
  DOUBLE PRECISION X(N), WORK(N)
  CALL C06GBF( X, N, IFAIL )
  CALL C06FBF( X, N, WORK, IFAIL )
  ```

  where X is in NAG format, with

  ```fortran
  DOUBLE PRECISION X(N), WORK(2*N+15)
  CALL PDA_DRFFTB( N, X, WORK )
  ```
where X is in FFTPACK format.

- Add calls to `PDA_DRFFT1` if necessary

  The work array supplied to `PDA_DRFFTB` needs initialising before calling `PDA_DRFFTB`. This is done by calling `PDA_DRFFT1`:

  ```fortran
  DOUBLE PRECISION WORK(2*N+15)
  CALL PDA_DRFFT1( N, WORK )
  ```

  There is no need to re-initialise WORK if the value of N has not changed since the previous call to `PDA_DRFFT1` (and if the contents of the work array have not been altered).

### 8.5 Replacing calls to C06FCF

C06FCF is the NAG routine for finding the FFT of a one-dimensional sequence of complex data values. The routine performs a forward transform. To do an inverse transform the complex conjugate of the input data is taken before calling C06FCF (using C06GCF), and the complex conjugate of the output data is taken on return from C06FCF.

The steps involved in replacing C06FCF calls with equivalent FFTPACK calls are listed separately for forward and inverse transforms.

#### 8.5.1 Forward transforms

- Re-organise the input (spatial domain) data

  The NAG routine expects real and imaginary parts in separate arrays, whereas FFTPACK expects them in the same array, with corresponding real and imaginary values in adjacent elements. If the application can be changed to supply the input data in this format, so well and good. Otherwise you will have to have an extra work array in which to hold the input (and output) data in FFTPACK format. You would convert the supplied input data using code such as:

  ```fortran
  DOUBLE PRECISION X(N), Y(N), C(2*N)
  DO J = 1, N
    I = 2*J
    C(I - 1) = X(J)
    C(I) = Y(J)
  END DO
  ```

  or

  ```fortran
  DOUBLE PRECISION X(N), Y(N), C(2, N)
  DO J = 1, N
    C(1, J) = X(J)
    C(2, J) = Y(J)
  END DO
  ```

  where the X and Y arrays hold the supplied data, C is a work array, and N is the number of data points.
• **Increase the size of the work array**

The work array passed to the FFT routine needs to be increased in size from N elements (for NAG) to 4*N+15 (for FFTPACK).

• **Replace call to C06FCF with PDA_DCFFTF**

Replace the call

```fortran
DOUBLE PRECISION X(N), Y(N), WORK(N)
CALL C06FCF( X, Y, N, WORK, IFAIL )
```

with

```fortran
DOUBLE PRECISION C(2*N), WORK(4*N+15)
CALL PDA_DCFFTF( N, C, WORK )
```

• **Add calls to PDA_DCFFTI if necessary**

The work array supplied to PDA_DCFFTF needs initialising before calling PDA_DCFFTF. This is done by calling PDA_DCFFTI:

```fortran
DOUBLE PRECISION WORK( 4*N+15 )
CALL PDA_DCFFTI( N, WORK )
```

There is no need to re-initialise WORK if the value of N has not changed since the previous call to PDA_DCFFTI (and if the contents of the work array have not been altered). No harm will occur (except for significant slowing down of execution) if the WORK array is unnecessarily re-initialised, but it is a good idea to include some logic to prevent this.

• **Convert output (frequency domain) data to NAG format**

The Fourier coefficients created by FFTPACK are stored in a single array and are not normalised, whereas NAG stores them in two arrays and normalises them. You can either modify the way your application to use the FFTPACK format instead of the NAG format, or call the PDA_DC2NAG routine to convert the FFTPACK results into NAG format.

```fortran
DOUBLE PRECISION X(N), Y(N), C(2*N)
CALL PDA_DC2NAG( N, C, X, Y )
```

where C is the output from PDA_DCFFTF, and X and Y hold the corresponding real and imaginary coefficients as returned by C06FCF.

### 8.5.2 Inverse transforms

• **Convert input (frequency domain) data to FFTPACK format**

If you choose not to modify your application to use FFTPACK data format throughout, you can instead do all the conversions just before (and after) calling the FFTPACK routines. So, if your application supplied frequency domain data in NAG format, first convert it to FFTPACK format using the PDA_DNAG2C routine:

```fortran
DOUBLE PRECISION X(N), Y(N), C(2*N)
CALL PDA_DNAG2C( N, X, Y, C )
```
where C is an additional work array used to hold the FFTPACK format data, ready for use by PDA_DCFFTB. X and Y are the supplied frequency domain data in NAG format. If this call to PDA_DC2NAG is made, the values returned by PDA_DCFFTB will have the same normalisation as the original data supplied to PDA_DCFFTF.

- **Increase the size of the work array**
  The work array passed to the FFT routine needs to be increased in size from N elements (for NAG) to 4*N+15 (for FFTPACK).

- **Replace call to C06FCF and C06GCF with PDA_DCFFTB**
  Using NAG, the inverse transform is usually done by the three calls:

  ```
  CALL C06GCF( Y, N, IFAIL )
  CALL C06FCF( X, Y, N, WORK, IFAIL )
  CALL C06GCF( Y, N, IFAIL )
  ```

  These three calls should be replaced by the single call:

  ```
  CALL PDA_DCFFTB( N, C, WORK )
  ```

  where C is the array into which the X and Y arrays have been converted using the method of the previous section.

- **Add calls to PDA_DCFFTI if necessary**
  The WORK array passed to PDA_DCFFTB should be initialised using PDA_DCFFTI before calling PDA_DCFFTB. Once the array has been initialised it can be used in multiple calls to PDA_DCFFTF and PDA_DCFFTB so long as they all have the same value for N.

- **Re-organise the output (spatial domain) data**
  NAG puts the spatial domain results into two arrays (one real, one imaginary), whereas FFTPACK puts them into one. You can either modify your application to use the FFTPACK format or convert the FFTPACK results into NAG-style results using code such as:

  ```
  DOUBLE PRECISION X( N ), Y( N ), C( 2*N )
  DO J = 1, N
    I = 2*J
    X( J ) = C( I - 1 )
    Y( J ) = C( I )
  END DO
  ```

  or

  ```
  DOUBLE PRECISION X( N ), Y( N ), C( 2, N )
  DO J = 1, N
    X( J ) = C( 1, J )
    Y( J ) = C( 2, J )
  END DO
  ```
8.6 Replacing calls to C06FJF

C06FJF is the NAG routine for finding the FFT of an N-dimensional array of complex data values. The routine performs a forward transformation. To do an inverse transform the complex conjugate of the input data is taken before calling C06FJF (using C06GCF), and the complex conjugate of the output data is taken on return from C06FJF.

There are no equivalent routines in FFTPACK as found in NETLIB. PDA_DNFFTF and PDA_DNFFTB have been written, which do the equivalent of C06FJF. These routines are a bit different to genuine FFTPACK routines in that they do not need any initialisation, and use NAG format rather than native FFTPACK format for complex data arrays and Fourier coefficient arrays. Consequently, replacing C06FJF is a bit easier than replacing the one-dimensional routines.

The steps involved in replacing C06FJF calls with equivalent FFTPACK calls are listed separately for forward and inverse transforms.

8.6.1 Forward transforms

- **Increase the size of the work array**
  The work array passed to the FFT routine needs to be increased in size from 3*MAXDIM elements (for NAG) to 6*MAXDIM+15 (for FFTPACK). Here, MAXDIM is the size of the largest array dimension.

- **Replace call to C06FJF with PDA_DNFFTF**
  Replace the call

  ```
  DOUBLE PRECISION X(N), Y(N), WORK( 3*MAXDIM )
  INTEGER ND( NDIM )
  CALL C06FJF( NDIM, ND, N, X, Y, WORK, LWORK, IFAIL )
  ```

  with

  ```
  DOUBLE PRECISION X(N), Y(N), WORK( 6*MAXDIM + 15 )
  INTEGER ND( NDIM )
  CALL PDA_DNFFTF( NDIM, ND, X, Y, WORK, ISTAT )
  ```

  IF( ISTAT .NE. 0 ) THEN
    This means that NDIM was either less than 1 or greater than 20.
    Report a programming error!
  END IF

8.6.2 Inverse transforms

- **Increase the size of the work array**
  The work array passed to the FFT routine needs to be increased in size from 3*MAXDIM elements (for NAG) to 6*MAXDIM+15 (for FFTPACK). Here, MAXDIM is the size of the largest array dimension.

- **Replace call to C06FJF and C06GCF with PDA_DNFFTB**
  Using NAG, the inverse transform is usually done by the three calls:

```
CALL C06GCF(Y, N, IFAIL)
CALL C06FJF(NDIM, ND, N, X, Y, WORK, LWORK, IFAIL)
CALL C06GCF(Y, N, IFAIL)
These three calls should be replaced by the single call:
CALL PDA_DNFFTB(NDIM, ND, X, Y, WORK, ISTAT)

8.7 Replacing calls to C06FUF

C06FUF is the NAG routine for finding the FFT of a two-dimensional sequence of complex data values. There is no direct equivalent. Use the N-dimensional routines instead (with N = 2). See C06FJF.

8.8 Replacing calls to C06GBF and C06GCF

The complex conjugation NAG routines C06GBF and C06GCF should no longer be needed since separate routines are provided within FFTPACK for doing inverse transformation.

9 One-dimensional Interpolation and Fitting, Splines

The routines for this sort of application (and their origins) are:

- **PDA_BSPDOC** (SLATEC/CAMSUN)
  Documentation for BSPLINE, a package of subprograms for working with piecewise polynomial functions in B-representation.

- **PDA_DBINTK** (SLATEC/CAMSUN)
  Compute the B-representation of a spline which interpolates given data. The knots must be given.

- **PDA_DEFC** (SLATEC/CAMSUN)
  Fit a piecewise polynomial curve to discrete data. The piecewise polynomials are represented as B-splines. The fitting is done in a weighted least squares sense.

- **PDA_DBVALU** (SLATEC/CAMSUN)
  Evaluate the B-representation of a B-spline at X for the function value or any of its derivatives.

- **PDA_DBSQAD** (SLATEC/CAMSUN)
  Compute the integral of a K-th order B-spline using the B-representation.

- **PDA_CURFIT** (DIERCKX/NETLIB)
  Determine a smooth spline approximation of degree k to the given set of data points. The knots can be given, or can be determined by the routine.
• **PDA_SPLEV** (DIERCKX/NETLIB)
  Evaluates in a number of points x(i) a spline s(x) of degree k, given in its B-spline representation.

• **PDA_SPLDER** (DIERCKX/NETLIB)
  Evaluates in a number of points x(i) the derivative of order NU of a spline s(x) of degree k, given in its B-spline representation.

• **PDA_SPLINT** (DIERCKX/NETLIB)
  Calculates the integral of a spline function s(x) of degree k, which is given in its normalised B-spline representation.

• **PDA_DPLINT** (SLATEC/CAMSUN)
  Produce the polynomial which interpolates a set of discrete data points.

• **PDA_DPOLVL** (SLATEC/CAMSUN)
  Calculate the value of a polynomial and its first NDER derivatives where the polynomial was produced by a previous call to PDA_DPLINT.

• **PDA_DPOLCF** (SLATEC/CAMSUN)
  Compute the coefficients of the polynomial fit (including Hermite polynomial fits) produced by a previous call to PDA_DPLINT.

• **PDA_DPOLFT** (SLATEC/CAMSUN)
  Fit discrete data in a least squares sense by polynomials in one variable. Uses weights.

• **PDA_DPTVLU** (SLATEC/CAMSUN)
  Use the coefficients generated by PDA_DPOLFT to evaluate the polynomial fit of degree L, along with the first NDER of its derivatives, at a specified point.

• **PDA_DPCOEF** (SLATEC/CAMSUN)
  Convert the PDA_DPOLFT coefficients to Taylor series form.

### 9.1 B-splines

E01BAF finds the interpolating cubic spline interpolant f(x) for a set of points (x,y). The interpolant is evaluated with E02BBF, evaluated with derivatives by E02BCF, and integrated by E02BDF. PDA_DBINTK does this, the order of the splines can be changed as well. This routine needs to be given the knots, while E01BAF set them itself. Evaluation of the interpolant and its derivatives is done by PDA_DBVALU, integration by PDA_DBSQAD.

E02BAF finds the fitting cubic spline f(x) for a set of points and weights (x,y,w). The interior knots 5 ... n+3 must be given and are fixed. The function is evaluated with E02BBF, with derivatives by E02BCF, and integrated with E02BDF. PDA_DEFUC does this. All knots must be given, not just the interior ones. Instead of weights PDA_DEFUC takes standard deviations (x,y,sigma) and uses 1/sigma as weight. Evaluation of the function and its derivatives is done by PDA_DBVALU, integration by PDA_DBSQAD.

E02BBF and E02BCF evaluate an interpolating or fitting cubic spline and its derivatives. They follow a call to E01BAF or E02BAF. This function is taken over by PDA_DBVALU. E02BDF integrates an interpolating or fitting cubic spline. It follows a call to E01BAF or E02BAF. This function is taken over by PDA_DBSQAD.
E02BEF finds the fitting cubic spline \( f(x) \) for a set of points and weights \((x,y,w)\). The knots are located automatically. The function is evaluated with E02BBF, with derivatives by E02BCF, and integrated with E02BDF. PDA_CURFIT (from the DIERCKX package) solves this problem. While the other routines are from SLATEC and for double precision, PDA_CURFIT is for single precision. Hence, PDA_SPLEV, PDA_SPLDER and PDA_SPLINT should be used to evaluate the spline, its \( n \)-th derivative, and its integral.

### 9.2 Ordinary polynomials

E02ADF finds the fitting Chebyshev series minimising r.m.s. The Chebyshev series is equivalent to an ordinary polynomial, but cannot be extrapolated. The polynomial is evaluated by E02AEF or E02AKF. In the latter routine the Chebyshev coefficients can be one column of an array. It will also take the real-world \( x \) argument instead of the normalised \( x\)-bar argument within the range \(-1 \ldots +1\).

In this library, PDA_DPOLFT fits an ordinary polynomial as a sum of orthogonal polynomials. The representation returned is somewhat special. It can be converted to coefficients of a Taylor series with PDA_DPCOEF or directly evaluated with PDA_DP1VLU. PDA_DP1VLU will return in one call as many derivatives as requested.

### 9.3 Replacing calls to E01BAF

The SLATEC equivalent of this routine is **PDA_DBINTK** with order \( K = 4 \). The NAG code would look like

```fortran
  INTEGER M, IFAIL
  DOUBLE PRECISION X(M), Y(M), T(M+4), C(M+4), WRK(6*M+16)
  IFAIL = 1
  CALL E01BAF( M, X, Y, T, C, M+4, WRK, 6*M+16, IFAIL )
  IF ( IFAIL .NE. 0 ) THEN
      An error has occurred
  END IF
```

While the size of the coefficient vector can be reduced to \( M \) for PDA_DBINTK, you now need two work spaces. The major difference is that PDA_DBINTK needs to be given the knots. So you have to calculate them in the same way as E01BAF would have done. The handling of the status is different.

```fortran
  INTEGER I, K, M, IFAIL
  PARAMETER ( K = 4 )
  DOUBLE PRECISION X(M), Y(M), T(M+K), C(M)
  DOUBLE PRECISION WRK1( (2*K-1)*M ), WRK2( 2*K )
  DO 1 I = 1, K
      T(I) = X(1)
      T(M+I) = X(M)  ! Note: K is even
 1 CONTINUE
  DO 2 I = K+1, M
      T(I) = X(I-K/2)
 2 CONTINUE
  IFAIL = 0
```
CALL PDA_DBINTK( X, Y, T, M, K, C, WRK1, WRK2, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF

9.4 Replacing calls to E02BAF

PDA_DEFC has not yet been used anywhere to replace E02BAF. Thus the migration hints given here may contain errors or may be based on misunderstandings.

The SLATEC equivalent of this routine is [PDA_DEFC] with order K = 4. The NAG code would look like

```
INTEGER M, N, IFAIL
DOUBLE PRECISION X(M), Y(M), W(M), T(N+7), C(N+7), SS
DOUBLE PRECISION WORK1(M), WORK2( 4*(N+7) )
IFAIL = 1
CALL E02BAF( M, N+7, X, Y, W, T, WORK1, WORK2, C, SS, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
```

Here N+7 is the number of knots. Since the order is 4 (cubic), the number of interior knots is then N+7−8 = N−1. N is the number of intervals. The interior knots are T(5) ... T(N+3). The weights W are reciprocal errors of Y. Although C has a length of N+7 only N+3 coefficients are returned by E02BAF.

The interior knots T(5) ... T(N+3) are given arguments, but the remaining knots are set by E02BAF and thus returned arguments.

In PDA_DEFC the order is K = 4. There are N+K+3 knots T(). T(1) ... T(K−1) and T(N+5) ... T(N+K+3) are end knots. The next inner knots T(K) and T(N+4) are presumably the first and last x value. Then T(K+1) ... T(N+3) would be truly interior knots just as in the NAG code.

PDA_DEFC does not generate knots by itself. Contrary to the NAG code above, the first K and last K knots must be calculated before the call.

The size of the work space is more complex to calculate. PDA_DEFC needs the standard deviation in Y instead of the weights SD = 1/W. PDA_DEFC returns a diagnostic J, which should have value J = 1 if no error occurred.

```
INTEGER I, J, K, L, M, N, IFAIL
PARAMETER ( K = 4 )
PARAMETER ( L = (N+6) * (K+1) + (N+K+4) * (K+1)
   : + 2*MAX(M,N+K+3) + N+K+3 + K**2 )
DOUBLE PRECISION X(M), Y(M), W(M), SD(M)
DOUBLE PRECISION T(N+K+3), C(N+3)
DOUBLE PRECISION WORK(L)
DO 1 I = 1, K
   T(I) = MIN( X() )
   T(N+3+I) = MAX( X() )
1  CONTINUE
```
DO 2 I = 1, M
   SD(I) = 1D0 / W(I)
2 CONTINUE
IFAIL = 0
CALL PDA_DEFC( M, X, Y, SD, K, N+K+3, T, 1, J, C, L, WORK, IFAIL )
IF ( J .NE. 1 .OR. IFAIL .NE. 0 ) THEN
   An error has occurred
END IF

9.5 Replacing calls to E02BBF

The equivalent of this routine in SLATEC is PDA_DBVALU with the requested derivative being zero and the order being $K = 4$ for a cubic spline. The NAG code would look like

```fortran
INTEGER N, IFAIL
DOUBLE PRECISION T(N+7), C(N+7), X, S
IFAIL = 1
CALL E02BBF( N+7, T, C, X, S, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
```

Here $N$, $T$ and $C$ are the same as in E02BAF. If $T$ and $C$ originate from a call to E01BAF then for $N+7$ read $M+4$ with $M$ the number of data points given to the interpolation.

PDA_DBVALU is a function rather than a subroutine. The dimension passed to PDA_DBVALU is not that of $T$, but that of $C$, i.e. $N+3$ (or $M$ after interpolation). PDA_DBVALU returns the value of any derivative, the fifth argument is zero so that it returns the function value itself. INVVB must be given 1 in the first call. For several evaluations of the same spline it should not be changed between calls. It is changed by PDA_DBVALU. So if PDA_DBVALU is called in a DO loop, the statement INVVB = 1 is typically before and outside the loop.

```fortran
INTEGER INVVB, K, N
PARAMETER ( K = 4 )
DOUBLE PRECISION T(N+K+3), C(N+3), X, S
DOUBLE PRECISION WORK(3*K)
DOUBLE PRECISION PDA_DBVALU
INVVB = 1
IFAIL = 0
S = PDA_DBVALU( T, C, N+3, K, 0, X, INVVB, WORK, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
```

9.6 Replacing calls to E02BCF

E02BCF has not yet been replaced anywhere. Thus the migration hints given here may contain errors or may be based on misunderstandings.

The equivalent of this routine in SLATEC is PDA_DBVALU. Several calls are necessary, one for each derivative. For the function value itself set the number of derivative to zero. The order is $K = 4$ for a cubic spline. The NAG code would look like

```fortran
INTEGER INVVB, K, N
PARAMETER ( K = 4 )
DOUBLE PRECISION T(N+K+3), C(N+3), X, S
DOUBLE PRECISION WORK(3*K)
DOUBLE PRECISION PDA_DBVALU
INVVB = 1
IFAIL = 0
S = PDA_DBVALU( T, C, N+3, K, 0, X, INVVB, WORK, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
```
Here N, T and C are the same as in E02BAF. If T and C originate from a call to E01BAF then for N+7 read M+4 with M the number of data points given to the interpolation. S(I) returns the I-th derivative. In the case that X coincides with a knot and the derivatives are not continuous at that knot, LEFT is used to decide which side of the knot to use.

PDA_DBVALU is a function rather than a subroutine. The dimension passed to PDA_DBVALU is not that of T, but that of C, i.e. N+3 (or M after interpolation). PDA_DBVALU returns the value of any derivative, as specified in the fifth argument.

There is no equivalent to the LEFT parameter in NAG. PDA_DBVALU returns right limiting values, except at the right end point.

INVB must be given 1 in the first call. For several evaluations of the same spline it should not be changed between calls. It is changed by PDA_DBVALU. So if PDA_DBVALU is called in a DO loop, the statement INVB = 1 is typically before and outside the loop. In the code below, IFAIL is reset inside the DO loop. Assuming that an error will quit the loop, the IFAIL = 0 statement could be before and outside the DO loop as well.

9.7 Replacing calls to E02BDF

PDA_DBSQAD has not yet been used anywhere to replace E02BDF. Thus the migration hints given here may contain errors or may be based on misunderstandings.

The equivalent of this routine in SLATEC is PDA_DBSQAD. The NAG code would look like
Here \( N, T \) and \( C \) are the same as in \texttt{E02BAF}. If \( T \) and \( C \) originate from a call to \texttt{E01BAP}, then for \( N+7 \) read \( M+4 \) with \( M \) the number of data points given to the interpolation. \( \text{DEFINT} \) returns the integral over the whole \( x \) range where the spline is defined. This is from \( T(4) \) to \( T(N+4) \), which are most probably the smallest and largest \( X \) used in the fit or interpolation.

The dimension passed to \texttt{PDA_DBSQAD} is not that of \( T \), but that of \( C \), i.e. \( N+3 \) (or \( M \) after interpolation). \texttt{PDA_DBSQAD} calculates the integral for any interval on which the spline is defined. For the same interval as in the NAG code, the two limiting knots are given to \texttt{PDA_DBSQAD}.

\begin{verbatim}
INTEGER K, N
PARAMETER ( K = 4 )
DOUBLE PRECISION T(N+K+3), C(N+3), DEFINT
DOUBLE PRECISION WORK(3*K)
CALL PDA_DBSQAD( T, C, N+3, K, T(4), T(N+4), DEFINT, WORK, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
\end{verbatim}

### 9.8 Replacing calls to \texttt{E02BEF}

\texttt{E02BEF} is a more advanced routine than the other NAG spline routines. It places knots automatically while fitting a cubic spline. This library includes for this case a number of routines from DIERCKX. \texttt{PDA_CURFIT} performs the spline approximation with given or automatic knots. The fitted function is evaluated with \texttt{PDA_SPLEV}, its derivatives with \texttt{PDA_SPLDER}, its integral with \texttt{PDA_SPLINT}. The routines exist only for single precision arguments.

These routines are so far unused, so there are no migration hints.

### 9.9 Replacing calls to \texttt{E02ADF}

Superficially, the equivalent SLATEC routine is \texttt{PDA_DPOLFT}. Since NAG has no routine to do a polynomial extrapolation, Figaro usurped this routine with a peculiar set of weights and very few degrees of freedom to extrapolate a polynomial. Although \texttt{PDA_DPOLFT} can be used even in that case, it is \texttt{PDA_DPLINT} that is intended for polynomial interpolation.

The NAG code would look like

\begin{verbatim}
INTEGER I, M, K, NROWS, IFAIL
DOUBLE PRECISION X(M), Y(M), W(M)
DOUBLE PRECISION WORK1(3*M), WORK2( 2*(K+1) )
DOUBLE PRECISION A1(NROWS,K+1), S(K+1)
DOUBLE PRECISION A2(K+1)
IFAIL = 1
CALL E02ADF( M, K+1, NROWS, X, Y, W, WORK1, WORK2, A1, S, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
   An error has occurred
END IF
DO 1 I = 1, K+1
   A2(I) = A1(K+1,I)
1 CONTINUE
\end{verbatim}
Here W are the weights proportional to the reciprocal of the standard deviation of Y. A1 returns a matrix of Chebyshev coefficients, one set of coefficients for each polynomial degree from 0 to K. The DO loop extracts the coefficients for degree K into A2. Note that these coefficients form a column rather than a row in A1. S returns the r.m.s. for the fit of each degree from 0 to K.

The behaviour of PDA_DPOLFT is controlled by the given value of EPS, passing zero (0D0) makes it perform fits for all degrees from 0 to K. EPS is also a returned argument, it returns the r.m.s. for the highest degree fitted. What degree that was is returned in NDEG. An indication of the success is returned in IFAIL1.

The weights should be proportional to the reciprocal of the variance, i.e. the square of the weights used for NAG.

The returned description of the polynomials A3 is rather different from the Chebyshev coefficients returned by E02ADF. A3 would be passed on to PDA_DP1VLU to evaluate the polynomial or to PDA_DPCOEF to convert A3 to coefficients of a Taylor series. A3 contains sufficient information to evaluate the polynomial of any degree from 0 to K. The desired degree is specified to PDA_DP1VLU or PDA_DPCOEF.

R is a returned vector in which the fit of highest degree is evaluated at all given X. Often this may render any further evaluation calls obsolete.

```fortran
INTEGER I, M, K, NDEG, IFAIL1, IFAIL2
DOUBLE PRECISION X(M), Y(M), W(M), W2(M), R(M)
DOUBLE PRECISION A3( 3*M + 3*(K+1) )
DOUBLE PRECISION EPS, S(K+1)
DO 1 I = 1, M
   W2(I) = W(I) * W(I)
1 CONTINUE
IFAIL2 = 0
EPS = 0D0
CALL PDA_DPOLFT( M, X, Y, W2, K, NDEG, EPS, R, IFAIL1, A3, IFAIL2 )
IF ( NDEG .NE. K .OR. IFAIL1 .NE. 1 .OR. IFAIL2 .NE. 0 ) THEN
   An error has occurred
END IF
DO 2 I = 1, K
   S(I) = 0D0
2 CONTINUE
S(K+1) = EPS
```

9.10 Replacing calls to E02AEF

The equivalent of this routine in SLATEC is PDA_DP1VLU. The NAG code would look like

```fortran
INTEGER I, K, K2, NROWS, IFAIL
DOUBLE PRECISION A1(NROWS,K+1)
DOUBLE PRECISION A2(K2+1), XX, XCAP, P
DO 1 I = 1, K2+1
   A2(I) = A1(K2+1,I)
1 CONTINUE
   XCAP = ( ( XX - MIN(X()) ) - ( MAX(X()) - XX ) )
   :     / ( MAX(X()) - MIN(X()) )
```
Here A1 is the coefficient matrix returned by E02ADF, A2 is the column extracted for the required degree. XX is the x value for which an evaluation is required, it must be scaled into the range \(-1 \ldots +1\), using the original extreme x values passed to E02ADF. P returns the function value.

PDA_DP1VLU returns any number of derivatives in addition to the function value. The second argument specifies how many derivatives are required. No scaling of XX is necessary, and no processing of the coefficients A3 as returned by PDA_DPOLFT.

9.11 Replacing calls to E02AKF

E02AKF is similar to E02AEF but provides a different interface. Namely, A1 (from E02ADF) and the row length can be given instead of the extraction A2. Also XX is given rather than the scaled XCAP. There are no detailed migration hints for this routine yet.

9.12 Replacing calls to GEN_CHB2NO

This is not a NAG routine, but a routine in Figaro’s GEN library. It converts the vector of Chebyshev coefficients into a vector of coefficients of an ordinary polynomial. This is particularly useful when these are to be written to output files. In future the routine will still be needed to read old files that contain Chebyshev coefficients.

This routine is mentioned here really, because it has an equivalent in SLATEC named PDA_DPCOEF. PDA_DPCOEF is more general, in that it returns coefficients for a Taylor series, i.e. a polynomial in \((XX-X0)\) for given expansion point X0.

The old code would look like

```
INTEGER I, K, K2, NROWS
DOUBLE PRECISION A1(NROWS,K+1)
DOUBLE PRECISION A2(K2+1), C(K2+1)
DO 1 I = 1, K2+1
   A2(I) = A1(K2+1,I)
1  CONTINUE
CALL GEN_CHB2NO( K2, MIN(X()), MAX(X()), A2, C )
```
This would be replaced by

```fortran
INTEGER K, K2, IFAIL
DOUBLE PRECISION A3( 3*M + 3*(K+1) )
DOUBLE PRECISION C(K2+1)
IFAIL = 0
CALL PDA_DPCOEF( K2, 0D0, C, A3, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
  An error has occurred
END IF
```

Just in case you are tempted to evaluate the polynomial yourself rather than use PDA_DP1VLU here is a piece of code to get $P = f(XX)$:

```fortran
INTEGER I, K2
DOUBLE PRECISION XX, P
DOUBLE PRECISION C(K2+1)
P = C(K2+1)
DO 2 I = K2, 1, -1
  P = P * XX
  P = P + C(I)
2 CONTINUE
```

## 10 Two-dimensional Interpolation and Fitting

The routines for this sort of application are:

- **PDA_CHE2D**
  PDA_CHE2D evaluates a 2-dimensional Chebyshev polynomial. A single precision version (PDA_CHE2R) is available.

- **PDA_BISPEV**
  PDA_BISPEV evaluates the bivariate spline approximation found by PDA_SURFIT.

- **PDA_DB2INK**
  PDA_DB2INK determines a piecewise polynomial function that interpolates the two-dimensional gridded data. Users specify the polynomial order (degree+1) of the interpolant and (optionally) the knot sequence.
  The interpolating function is a piecewise polynomial represented as a tensor product of one-dimensional B-splines.

- **PDA_DB2VAL**
  PDA_DB2VAL evaluates the tensor product piecewise polynomial interpolant constructed by the routine PDA_DB2INK, or, alternatively evaluates one of its derivatives, at a given point. Function values returned are double precision.

- **PDA_IDBVIP**
  PDA_IDBVIP performs bivariate interpolation when the data points are irregularly distributed in the x-y plane. Function values returned are single precision.

- **PDA_IDSFFT**
  PDA_IDSFFT performs smooth surface fitting when the data points are irregularly distributed in the x-y plane. Function values returned are single precision.
• **PDA_SURFIT** PDA_SURFIT determines a smooth bivariate spline approximation for irregularly distributed data.

E02DAF calculates a 2-D bi-cubic spline interpolating surface for points from a regular grid. The routines E02DEF/E02DFF may then be used to compute values of the spline at the required location. This approach has been replaced by using the routine pair PDA_DB2INK and PDA_DB2VAL. No significant change in program performance was found.

E02SAF provides a 2-D surface fit for data on an irregular spaced grid. The method employed is that of Renka and Cline where the grid is used to construct a set of suitably weighed equiangular triangles that (with appropriate weighting) describe the surface. The interpolated value of the gridded data at any point within the grid is extracted by E02SBF. This pair of routines has been replaced using PDA_IDBVIP with no significant change in program performance. During incomplete trials the routine PDA_IDSFFT also appeared to give sensible results.

### 10.1 Replacing calls to E02DAF

E02DAF can be replaced with the following code.

* Declare variables
  
  INTEGER ID ! Specifies use value not differential
  INTEGER IFAIL ! Was the surface successfully created?
  INTEGER MXY ! Size of the grid
  INTEGER ORD ! Order of polynomial used
  DOUBLE PRECISION DVALUE ! Interpolated value returned
  DOUBLE PRECISION FV1(8,8) ! Grid Z values
  DOUBLE PRECISION X1(8), Y1(8) ! X,Y grid locations
  DOUBLE PRECISION XD, YD ! X,Y coord for interpolation
  DOUBLE PRECISION BCOEF(8,8) ! Array used by PDA_DB2INK
  DOUBLE PRECISION TX(11),TY(11) ! Array used by PDA_DB2INK
  DOUBLE PRECISION WORK(168) ! Array used by PDA_DB2INK

  * Subroutine initial error values.
    IFAIL=0
    STATUS=0

  * Order of polynomial used in this example.
    ORD=3

  * Use value rather than differential.
    ID=0

  * Size of data grid. 8x8 in this instance.
    MXY=8

  * Build the surface fit using the grid contents.
    CALL PDA_DB2INK(X1,MXY,Y1,MXY,FV1,MXY,
    : ORD,ORD,TX,TY,BCOEF,
    : WORK,IFAIL,STATUS)
* If IFAIL=1 then okay to interpolate values.
  IF (IFAIL.EQ.1) THEN
  * Setup evaluation routine.
    IFAIL=0
    CALL PDA_DB2VAL(XD,YD,ID,ID,TX,TY,
    : MXY,MXY,ORD,ORD,BCOEF,WORK,
    : DVALUE,IFAIL,STATUS)
  END IF

In the example shown the value of the surface fitted is calculated at a point defined by XD and YD. The surface constructed is a 3rd order polynomial and the value returned by PDA_DB2VAL is the surface value not its differential. The example assumes that the array FV1() already contains values for the data at each of the grid points defined in arrays X1 and Y1. The size of the work arrays is determined by the size of the grid required. The variable DVALUE contains the interpolated value.

10.2 Replacing calls to E02SAF

It has been found in tests that the following code examples adequately replace calls to E02SAF.

* Local Variables:
  INTEGER ISTAT ! Status
  INTEGER MD ! Mode
  INTEGER NDP ! Grid points
  INTEGER NCP ! Not used
  INTEGER NOP ! Size of returned array
  INTEGER IWK(2500) ! Workspace
  REAL WK(640) ! Workspace
  REAL XD(80),YD(80),ZD(80) ! Surface data
  REAL XI(1),YI(1) ! Extrap points
  REAL ZI(1,1) ! Results

* Set the error flag default values.
  ISTAT=0
  STATUS=0

* Set mode and the grid positions (for PDA_IDBVIP).
  MD= 1
  NCP=2
  NOP=1

* Call interpolation subroutine.
  CALL PDA_IDBVIP(MD,NCP,NDP,XD,YD,ZD,NOP,XI,YI,
  : ZI,IWK,WK,ISTAT,STATUS)

In the example shown, the value of the interpolated surface at XI(1),YI(1) is returned in the variable ZI(1,1). The surface is constructed using co-ordinate information from the arrays
of XD() and YD(), and surface values contained in the array ZD(). The number of data points available to define the surface is NDP.

If several values are required from locations within the irregular grid, the mode variable MD should for the first point be set to 1 but may subsequently be 2. This significantly increases the execution speed. The size of the work arrays is determined by the size of the grid required.

Alternatively, the routine PDA_IDSFFT may be used thus:

```plaintext
INTEGER ISTAT ! Status
INTEGER MD ! Mode
INTEGER NDP ! Grid points
INTEGER NCP ! Not used
INTEGER NXI,NYI ! Output grid size
INTEGER IWK(2500) ! Workspace
REAL XD(80),YD(80),ZD(80) ! Surface data
REAL XI(1),YI(1) ! Extrap points
REAL ZI(1,1) ! Results
REAL WK(640) ! Workspace

* Set the error flag default values.
  ISTAT=0
  STATUS=0

* Set up the grid positions.
  NXI=1
  NYI=1

* Set mode and the grid positions (for PDA_IDBVIP).
  MD= 1
  NCP=2

* Call the surface fitting subroutine.
  CALL PDA_IDSFFT(MD,NCP,NDP,XD,YD,ZD,
      : NXI,NYI,XI,YI,ZI,IWK,
      : WK,ISTAT,STATUS)
```

In the example shown the surface is constructed from the irregular grid information contained in arrays XD() / YD() (location) and ZD() (surface value). Interpolated values calculated from the fitted surface are returned in the ZI() array at the co-ordinates specified by the data in the XI() and YI() arrays.

If several values are required from locations within the irregular grid, the mode variable MD should for the first point be set to 1 but may subsequently be 2. This significantly increases the execution speed. The size of the work arrays is determined by the size of the grid required.

### 10.3 Replacing calls to E02CBF

The routine PDA_CHE2D (or its single precision equivalent PDA_CHE2R) can be used to replace E02CBF. The replacement is straightforward with most of the arguments being the same, albeit in a slightly different order.
11 Minimisation

The routines for minimisation (or optimisation) in this library are:

- **PDA_DNLS1**
  Minimises the sum of squares of M non-linear functions.

- **PDA_DNLSIE**
  Minimises the sum of squares of M non-linear functions (easy version).

- **PDA_QQED**
  Solve bounded nonlinear least squares and nonlinear equations.

- **PDA_LMDIF1 (MINPACK/NETLIB)**
  Minimise the sum of the squares of m nonlinear functions in n variables, simple interface to PDA_LMDIF.

- **PDA_LMDIF (MINPACK/NETLIB)**
  Minimise the sum of the squares of m nonlinear functions in n variables with a modified Levenberg-Marquardt algorithm. Needs function only, the Jacobian is calculated by a forward-difference approximation.

- **PDA_UNCMND (NMS/TIBER)**
  Minimises a smooth non-linear function of n variables. Needs function values only.

- **PDA_SA (module SIMANN from OPT/NETLIB)**
  Continuous simulated annealing global optimisation algorithm. Simple constraints can be specified.

- **PDA_SUMSL (module SUMSL from TOMS)**
  Minimises a general unconstrained objective function using analytic gradients and a hessian approximation from secant update.

- **PDA_SUBPLX (module SUBPLEX from OPT/NETLIB)**
  Subplex method to solve unconstrained optimisation problems. The method is well suited for optimising objective functions that are noisy or are discontinuous at the solution.

11.1 Overview

There are two sorts of minimisation, one is to minimise any old function, and one to minimise a sum of squares. The first is more general, the least-squares routines are presumably more efficient. For the programmer the differences are as follows:

- For a least-squares fit, your merit function is a vector of residuals between measurements and current model guess. For a general minimisation your merit function is a scalar, basically you have to add up the squared residuals within your merit function.
• Perhaps more important is the amount of workspace you have to provide to the fit algorithm. In least-squares fits this scales with \( n \times m \) where \( n \) is the number of parameters to be fitted and \( m \) is the number of residuals to be added up (e.g. channels in a spectrum). \( m \) is quite large and depends on the data set at hand. The general minimisation does in principle not know that there is a spectrum with \( m \) pixels, and its workspace scales with the square of \( n \), which is more or less constant for any given application.

• Another difference is that least-squares fits tend to return the Jacobi matrix, the derivatives of each fit parameter with respect to each measurement. This is quite valuable if you want to know the variances of the fit parameters and the covariances between them. With the general minimisation you would have to work out the Hesse matrix and invert it. Which means you have to be able to work out the derivatives of you merit function w.r.t. to each fit parameter.

Two other issues in choosing minimisation algorithms are whether only the merit function can be provided or also first derivatives, and whether the fit parameters have to be constrained or not. It appears that Starlink applications use mainly unconstrained fits or at most simple bounds, i.e. hard constant limits on parameters. This library contains function-only algorithms, and also the SUMSL algorithm which requires both functions and gradients. The only constrainable algorithm is the simulated annealing SIMANN.

• PDA_UNCMND is a general unconstrained minimisation using function values only. A quasi-Newton algorithm with line search is used.

• PDA_LMDIF/PDA_LMDIF1 is an unconstrained least-squares minimisation using residuals only (no derivatives). A modified Levenberg-Marquardt algorithm is used, the Jacobian is calculated by a forward-difference approximation.

• SIMANN is a simulated annealing algorithm. It uses function values only and can be used for non-smooth functions as well. It should also have a fair chance of getting out of local minima and going on to find the global minimum.

• SUMSL is a general unconstrained minimisation using function values and gradients. A trusted regions algorithm is used.

• SUBPLEX is a generalisation and improvement on the Simplex algorithm. It should be very robust with any function to minimise. But it should also be rather inefficient.

11.2 Replacing calls to E04DGF and E04DKF

E04DGF performs an unconstrained minimisation using function values and first derivatives, E04DKF is just an auxiliary routine. To replace these, PDA_UNCMND would be used, which does not make use of derivatives.

The existing NAG code might look as shown below. Two modules are involved, one controls the NAG fit routine, the other serves it by providing the value and gradient of the merit function (the function to be minimised). Information is passed to the merit function in two ways. Scalars and constant-size arrays are in a common block, while one variable-length array is passed as the USER argument. Its length is passed in IUSER. The controlling function may also call the merit function directly.
SUBROUTINE DOAFIT( ... )
INTEGER N
INTEGER IUSER
INTEGER ITER, IFAIL
INTEGER IWORK(N+1)
REAL USER(IUSER)
DOUBLE PRECISION X(N), FVAL, FGRAD(N)
DOUBLE PRECISION WORK(13*N)
EXTERNAL MERIT
+- COMMON / ABLOCK / constant-size arrays
| CALL MERIT( 0, N, X, FVAL, FGRAD, 0, IUSER, USER )
| IFAIL = 1
| CALL E04DGF( N, MERIT, ITER, FVAL, FGRAD, X, IWORK, WORK,
| : IUSER, USER, IFAIL )
| IF ( IFAIL .NE. 0 ) THEN
| An error has occurred
| END IF
| END

SUBROUTINE MERIT( MODE, N, X, FVAL, FGRAD, NSTATE, IUSER, USER )
INTEGER MODE, N, NSTATE
INTEGER IUSER
REAL USER(IUSER)
DOUBLE PRECISION X(N), FVAL, FGRAD(N)
+- COMMON / ABLOCK / constant-size arrays
FVAL = ...
DO 1 I = 1, N
  FGRAD(I) = ...
1 CONTINUE
END

PDA_UNCMND has separate arguments for the guess and the fit result. With PDA_UNCMND
the merit function is simpler in that it need not calculate the gradient. It also has a simpler
interface and cannot be passed a variable-length array as above. Thus a pointer to such an array
must be passed in the common block. In order to ease de-referencing this pointer, the merit
function is split into two modules. MERIT1 receives the passed arguments from its caller and it
receives the common block from the master routine DOAFIT. That apart MERIT1 does nothing
but call MERIT2. In this call the array pointer is de-referenced.

SUBROUTINE DOAFIT( ... )
INTEGER N
INTEGER IUSER, POINTR
INTEGER ITER, IFAIL
INTEGER IWORK(N+1)
REAL USER(IUSER)
DOUBLE PRECISION GUESS(N), FIT(N), FVAL
DOUBLE PRECISION WORK( N*(N+10) )
EXTERNAL MERIT1
+- COMMON / BBLOCK / constant-size arrays,
| : IUSER, POINTR
| POINTR = %LOC(USER)
| CALL MERIT1( N, GUESS, FVAL )
| IFAIL = 0
An alternative to a common block is to have a subroutine with SAVE variables as a reservoir. One routine can call the reservoir routine to set values and another can call it to retrieve values. %LOC and %VAL are not standard Fortran 77. The only way around it would be to use a different programming language such as Fortran 90 or C.

### 11.3 Replacing calls to E04FDF and E04GBF

These routines find the unconstrained minimum of a sum of squares. E04FDF uses function values only while E04GBF uses first derivatives as well. Either would be replaced by PDA_LMDIF or PDA_LMDIF1, which use only function values.

### 11.4 Replacing calls to E04HCF

This routine checks a user-supplied gradient function. It is obsolete when NAG is not used, especially when derivatives are not used by the minimisation algorithms.

### 11.5 Replacing calls to E04JAF and E04KDF

These are fairly general easy-to-use minimisations that allow simple bounds. E04JAF uses only function values while E04KDF also uses first derivatives. There is no direct equivalent in this library. PDA_UNCMND might be used in a number of cases. PDA_SUBPLX is a robust general algorithm but cannot be constrained. PDA_SA is the only constrainable algorithm in this library. Migration hints are not yet available.
11.6 Replacing calls to E04YCF

This routine is a follow-up to E04FCF, E04FDF and E04GBF to convert the Jacobian of a least-squares minimisation to the covariance matrix of the fitted parameters. See the documentation of E04YCF, of the minimisation routine used previously, and of PDA_LMDIF. Migration hints are not yet available.

12 Matrices

The routines for matrix operations in this library are:

- **PDA_DGEFA** (SLATEC/CAMSUN)
  Factor a matrix using Gaussian elimination. This is needed before the determinant and inverse can be calculated by PDA_DGEDI.

- **PDA_DGEDI** (SLATEC/CAMSUN)
  Compute the determinant and inverse of a matrix using the factors computed by PDA_DGECO or PDA_DGEFA.

- **PDA_DGEFS** (SLATEC/CAMSUN)
  Solve a general system of linear equations. This solves the problem $A \times x = b$. $A$ is a square matrix, $x$ and $b$ are vectors. The problem $A \times X = B$ where all are matrices can be solved as several systems $A \times x = b$. This routine supports such an undertaking since it is able to re-use a previous factorisation of $A$.

- **PDA_DBOLS** (SLATEC/CAMSUN)
  Solve the problem $E \times x = f$ (in the least squares sense) with bounds on selected $x$ values. $E$ is a matrix, $x$ and $f$ are vectors.

- **PDA_LSQR**
  Solves sparse unsymmetric, linear least squares and damped least squares problems

12.1 Overview

There is some excess baggage in this field in NAG, since it distinguishes approximate from accurate routines. The approximation is not in the analytical, but in the numeric sense.

Leaving F04QAF aside, the need is for:

- A matrix inverter (F04AAF is used only as an inverter).

- A solver for $A \times x = b$ where $A$ is square and $x$ and $b$ are vectors. The problem $A \times X = B$ where $A$ is $n$ by $n$, $X$ and $B$ are $n$ by $m$, can be split into $m$ problems $A \times x = b$ all with the same $A$.

- The third need is a least-squares solver for $A \times x = b$ where $A$ is not square and $b$ has more dimensions than $x$. 
Looking at SLATEC, four routines are needed. PDA_DGEDI computes the determinant and/or inverse of a matrix, but must be preceded by a factoriser, namely PDA_DGEFA. PDA_DGEFS solves $A \times x = b$ where $A$ is square. It can re-use a factorisation of $A$ from a previous run, and in that sense supports the solution of $A \times X = B$. Finally, PDA_DBOLS solves the over-determined problem $A \times x = b$ in the least-squares sense.

### 12.2 Replacing calls to F01AAF

This routine is an approximate matrix inverter and would be replaced by PDA_DGEFA followed by PDA_DGEDI. The NAG code might look like

```fortran
INTEGER IA, N, IX, IFAIL
DOUBLE PRECISION A(IA,N), X(IX,N), WORK(N)
IFAIL = 1
CALL F01AAF( A, IA, N, X, IX, WORK, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
  An error has occurred
END IF
```

The SLATEC routines invert the matrix in situ, so $A$ must be copied to $X$ first. It is important that PDA_DGEDI be called only if PDA_DGEFA signals correct processing as IFAIL = 0. Otherwise PDA_DGEDI will encounter a division by zero. The last argument to PDA_DGEDI determines the returned information, 1 chooses inverted matrix but no determinant.

```fortran
INTEGER I, J, IA, N, IX, IFAIL
INTEGER IPVT(N)
DOUBLE PRECISION A(IA,N), X(IX,N), WORK(N), DUMMY(2)
DO 2 J = 1, N
  DO 1 I = 1, N
    X(I,J) = A(I,J)
  1 CONTINUE
2 CONTINUE
CALL PDA_DGEFA( X, IX, N, IPVT, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
  An error has occurred
ELSE
  CALL PDA_DGEDI( X, IX, N, IPVT, DUMMY, WORK, 1 )
END IF
```

### 12.3 Replacing calls to F01ABF

This routine is a matrix inverter and would be replaced by PDA_DGEFA followed by PDA_DGEDI. Migration hints are not yet available.

### 12.4 Replacing calls to F04AAF

Although this routine solves $A \times X = B$, where all three are matrices, it is actually used only to find the inverse of a matrix. In that use all three matrices are square, $A$ is given, $B$ is unity, and $X$ is the inverse of $A$. The NAG code might look like
The SLATEC routines invert the matrix in situ, so A must be copied to X first. It is important that
PDA_DGEDI be called only if PDA_DGEFA signals correct processing as IFAIL = 0. Otherwise
PDA_DGEDI will encounter a division by zero. The last argument to PDA_DGEDI determines
the returned information, 1 chooses inverted matrix but no determinant.

12.5 Replacing calls to F04AEF

This routine solves A * X = B. The problem would be split into a number of problems A * x_i
= b_i, where x_i and b_i are corresponding columns of X and B. Each problem is solved by
PDA_DGEFS, which can re-use a factorisation of A that it worked out in the first call. Migration
hints are not yet available.

12.6 Replacing calls to F04ANF and F01AXF

F04ANF solves an over-determined problem A * x = b and would be replaced by PDA_DBOLS.
F01AXF is an auxiliary routine. Migration hints are not yet available.

12.7 Replacing calls to F04ASF and F04ATF

These routines solve A * x = b where A is square and would be replaced by PDA_DGEFS.
Migration hints are not yet available.
12.8 Replacing calls to F04QAF

The routine PDA_LSQR solves exactly the same problems as F04QAF, the only differences are that the workspace requirements and argument ordering are slightly different.

13 Sorting

- **PDA_DSORT (SLATEC/CAMSUN)**
  Sort an array and optionally make the same interchanges in an auxiliary array.

- **PDA_IPERM**
  Forms the inverse of a permutation.

- **PDA_QSAD**
  Sort a DOUBLE PRECISION array into ascending order.

- **PDA_QSAI**
  Sort an INTEGER array into ascending order.

- **PDA_QSAR**
  Sort a REAL array into ascending order.

- **PDA_QSDD**
  Sort a DOUBLE PRECISION array into descending order.

- **PDA_QSDI**
  Sort a INTEGER array into descending order.

- **PDA_QSDR**
  Sort a REAL array into descending order.

- **PDA_QSIAD**
  Sort an array of pointers to access a DOUBLE PRECISION array in ascending order.

- **PDA_QSIAI**
  Sort an array of pointers to access a INTEGER array in ascending order.

- **PDA_QSIAR**
  Sort an array of pointers to access a REAL array in ascending order.

- **PDA_QSIDD**
  Sort an array of pointers to access a DOUBLE PRECISION array in descending order.

- **PDA_QSIDI**
  Sort an array of pointers to access a INTEGER array in descending order.

- **PDA_QSIDR**
  Sort an array of pointers to access a REAL array in descending order.

- **PDA_RINP**
  Reorders an array in place using a permutation index.
• **PDA_SAAC**
  Sorts the columns of a two dimensional array into ascending order.

• **PDA_SAAR**
  Sorts the rows of a two dimensional array into ascending order.

### 13.1 Replacing calls to M01DJF

The routines PDA_SAACD solves the same problem as M01DJF, except that the columns of the array are sorted using an index vector rather than ranks. The index vector can be used to re-order other arrays (via the PDA_RINPD routine) or it can be permuted into a rank using PDA_IPERM. You should also take care to make sure that necessary extra elements are available in M, IP and the new workspace LINK.

### 14 Normal Distribution

• **PDA_DERF (SLATEC/CAMSUN)**
  Calculates the double precision error function for double precision argument X.

• **PDA_DERFC (SLATEC/CAMSUN)**
  Calculates the double precision complementary error function for double precision argument X.

• **PDA_PPND16**
  Returns the normal deviate corresponding to a given lower tail area of P.

The error function erf() in NAG is S15AEF. In SLATEC it is replaced by PDA_DERF. The NAG code might look like

```c
INTEGER IFAIL
DOUBLE PRECISION X, P
DOUBLE PRECISION S15AEF
IAFAIL = 1
P = S15AEF( X, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
    An error has occurred
END IF
```

The new code would be

```c
INTEGER IFAIL
DOUBLE PRECISION X, P
DOUBLE PRECISION PDA_DERF
IAFAIL = 0
P = PDA_DERF( X, IFAIL )
IF ( IFAIL .NE. 0 ) THEN
    An error has occurred
END IF
```
The complementary error function \( \text{erfc}() \) in NAG is S15ADF. In SLATEC it is replaced by PDA_DERFC. Migration hints are not yet available, but the replacement of calls to S15ADF with PDA_DERF should be very similar to replacing S15AEF with PDA_DERF.

15 Bessel Functions

- **PDA_DBESJ1** (SLATEC/CAMSUN)
  Calculates the double precision Bessel function of the first kind of order one for double precision argument \( X \).

  *PDA_DBESJ1 has not yet been used anywhere to replace S17AFF. Thus the migration hints given here may contain errors or may be based on misunderstandings.*

  The Bessel function \( J_1() \) in NAG is S17AFF. In SLATEC it is replaced by PDA_DBESJ1. The NAG code might look like

  ```
  INTEGER IFAIL
  DOUBLE PRECISION X, P
  DOUBLE PRECISION S17AFF
  IFAIL = 1
  P = S17AFF( X, IFAIL )
  IF ( IFAIL .NE. 0 ) THEN
    An error has occurred
  END IF
  ```

  The new code would be

  ```
  INTEGER IFAIL
  DOUBLE PRECISION X, P
  DOUBLE PRECISION PDA_DBESJ1
  IFAIL = 0
  P = PDA_DBESJ1( X, IFAIL )
  IF ( IFAIL .NE. 0 ) THEN
    An error has occurred
  END IF
  ```

16 Simple Statistics

- **PDA_NSCOR**
  Calculates the approximate expected values of normal order statistics.

- **PDA_V11**
  Calculates an approximation to the variance of the largest normal order statistic.

- **PDA_COVMAT**
  Approximates the covariance matrix of normal order statistics.

- **PDA_DCOV**
  Calculates the covariance matrix for a nonlinear data fitting problem
16.1 Replacing calls to G01DBF

The routine PDA_NSCOR calculates the same values as G01DBF, except that only $N_2$ values are returned, rather than $N$. This is since the values are symmetric and can therefore be simply derived.

16.2 Replacing calls to G01DCF

The routine PDA_COVMAT calculates the same statistics as G01DCF. To use it requires that you also supply the variance of the largest normal order statistic (see PDA_V11) and that you increase the space required for the variance array. This is now a full array of values rather than a packed array. The following code shows how to convert this to the same form as output by the NAG routine:

```
K = 1
DO 3 J = 1, N
   DO 4 L = 1, J
      NAGVEC( K ) = V( L, J )
      K = K + 1
   4 CONTINUE
3 CONTINUE
```

17 Pseudo-Random Numbers

The routines for creating pseudo-random numbers in this library all have a period of $2^{26}$ and 6–7 digits accuracy. They are based upon code by Ahrens, Dieter, & Grube. They use a multiplicative congruential generator which is certainly not the state of the art and may not be suitable for critical or sophisticated use.

The routines are:

- **PDA_RAND** (NETLIB/TOMS599)
  Returns uniform pseudo-random numbers in the range 0 to 1.

- **PDA_RNEXP** (NETLIB/TOMS599)
  Draws pseudo-random numbers from an exponential distribution.

- **PDA_RNGAM** (NETLIB/TOMS599)
  Draws pseudo-random numbers from a Gamma-function distribution.

- **PDA_RNNOR** (NETLIB/TOMS599)
  Draws pseudo-random numbers from a Normal distribution of specified mean and standard deviation.

- **PDA_RNPOI** (NETLIB/TOMS599)
  Draws pseudo-random numbers from a Poisson distribution of specified mean.

- **PDA_RNSED** (NETLIB/TOMS599)
  Sets the seed. This must be called before any of the other random-number routines.
17.1 Setting the seed (replacements for G05CBF and G05CCF)

Before any random numbers can be selected, a seed must be set using PDA_RNSED. The integer seed should satisfy the relationship

\[ \text{seed} = 4 \times k + 1 \]

where \( k \) is a non-negative integer. A fixed seed gives rise to a reproducible sequence of pseudo-random numbers.

For a non-repeatable sequence, there is no equivalent to NAG routine G05CCF because the system clock used to create the seed is not accessible portably in Fortran, and PDA is independent of other libraries. However, the following code has the desired effect.

```fortran
INTEGER SEED, STATUS, TICKS, PID
INCLUDE 'PRM_PAR'

CALL PSX_TIME( TICKS, STATUS )
CALL PSX_GETPID( PID, STATUS )
SEED = TICKS + PID
SEED = MOD( SEED, VAL__MAXI / 4 ) * 4 + 1
SEED = MOD( SEED, 2**28 )

CALL PDA_RNSED( SEED )
```

PSX_TIME returns the time in units of clock ticks since some arbitrary time. See SUN/121 for more details and linking instructions. The above code also permits storage of the chosen seed.

17.2 Data type of the random numbers

There is a major difference between the PDA random-number routines and those provided in the standard NAG library: in general the former are single-precision functions, whereas the latter are double precision. However, PDA_RNPOI and the corresponding G05DRF are both integer functions.

17.3 Replacements for G05CAF and G05DAF

Like G05CAF, PDA_RAND has a dummy argument demanded by the Fortran standard. It is convenient to set it to zero. Here is an example where two random numbers are drawn from a uniform distribution between 0 and 1. In this example a fixed seed is used, but you could use the computer’s clock to create a random seed (see Section 17.1).

```fortran
INTEGER SEED
EXTERNAL PDA_RAND
REAL PDA_RAND, VALUES( 2 )

* Use a fixed seed of 1.
SEED = 1
CALL PDA_RNSED( SEED )
```
* Obtain two random numbers from a uniform distribution between 0 and 1.
VALUE( 1 ) = PDA_RAND( 0.0 )
VALUE( 2 ) = PDA_RAND( 0.0 )

The EXTERNAL statement is recommended, although in many cases it will be unnecessary. To obtain in the range \([a,b]\) as provided by G05DAF, merely apply the following relationship.

\[
\text{random value} = (b - a) \times \text{PDA_RAND}(0.0) + a
\]

17.4 Replacement for G05DBF

PDA_RNEXP is only a partial replacement for G05DBF in that it computes pseudo-random numbers from \(e^{-x}\), whereas G05DBF uses the function \(\frac{1}{a}e^{-x/a}\). Thus its argument is also a dummy mandated by the Fortran standard.

17.5 Replacements for G05DDF, G05DRF, and G05FFF

The following code shows the remaining three routines in action.

```fortran
INTEGER SEED
EXTERNAL PDA_RNGAM, PDA_RNNOR, PDA_RNPOI
REAL PDA_RNGAM, PDA_RNNOR, PDA_RNPOI, VALUES( 3 )

* Use a fixed seed of 1001.
SEED = 1001
CALL PDA_RNSED( SEED )

* Obtain a random number from a Normal distribution of mean 4.2 and
* standard deviation 0.15
VALUE( 1 ) = PDA_RNNOR( 4.2, 0.15 )

* Obtain a random number from a Poisson distribution of mean 3.4.
VALUE( 2 ) = PDA_RNPOI( 3.4 )

* Obtain a random number from a Gamma-function distribution of mean
* 1.2.
VALUE( 3 ) = PDA_RNGAM( 1.2 )
```

Apart from the change of data type, calls to G05DDF can be replaced with PDA_RNNOR using the same arguments. PDA_RNPOI is in effect a renamed G05DRF.

PDA_RNGAM only has one argument—the mean—of the Gamma function, whereas G05FFF has a second scaling parameter similar in role to the \(a\) argument of G05DBF. G05FFF also generates a vector of pseudo-random numbers.
A User-callable routines
PDA_BISPEV
Evaluates the bivariate spline approximation found by PDA_SURFIT.

Origin:
DIERCKX / NETLIB
Subroutine pda_bispev evaluates on a grid \((x(i),y(j)),i=1,\ldots,mx; j=1,\ldots,my\) a bivariate spline \(s(x,y)\) of degrees \(kx\) and \(ky\), given in the b-spline representation.

**Calling Sequence:**

```fortran
  call pda_bispev( tx, nx, ty, ny, c, kx, ky, x, mx, y, my, z, wrk, lwrk, iwrk, kwrk, ier )
```

**Input Parameters:**

- \(tx\) : Real array, length \(nx\), which contains the position of the knots in the x-direction.
- \(nx\) : Integer, giving the total number of knots in the x-direction.
- \(ty\) : Real array, length \(ny\), which contains the position of the knots in the y-direction.
- \(ny\) : Integer, giving the total number of knots in the y-direction.
- \(c\) : Real array, length \((nx-kx-1)*(ny-ky-1)\), which contains the b-spline coefficients.
- \(kx,ky\) : Integer values, giving the degrees of the spline.
- \(x\) : Real array of dimension \((mx)\). Before entry \(x(i)\) must be set to the x co-ordinate of the i-th grid point along the x-axis. \(tx(kx+1)<=x(i-1)<=x(i)<=tx(nx-kx), i=2,\ldots,mx\).
- \(mx\) : Integer. On entry \(mx\) must specify the number of grid points along the x-axis. \(Mx \geq 1\).
- \(y\) : Real array of dimension \((my)\). Before entry \(y(j)\) must be set to the y co-ordinate of the j-th grid point along the y-axis. \(ty(ky+1)<=y(j-1)<=y(j)<=ty(ny-ky), j=2,\ldots,my\).
- \(my\) : Integer. On entry \(my\) must specify the number of grid points along the y-axis. \(My \geq 1\).
- \(wrk\) : Real array of dimension \(lwrk\). Used as workspace.
- \(lwrk\) : Integer, specifying the dimension of \(wrk\). \(lwrk \geq mx*(kx+1)+my*(ky+1)\).
- \(iwrk\) : Integer array of dimension \(kwrk\). Used as workspace.
- \(kwrk\) : Integer, specifying the dimension of \(iwrk\). \(Kwrk \geq mx+my\).

**Output Parameters:**

- \(z\) : Real array of dimension \((mx*my)\). On successful exit \(z(my*(i-1)+j)\) contains the value of \(s(x,y)\) at the point \((x(i),y(j)),i=1,\ldots,mx;j=1,\ldots,my\).
- \(ier\) : Integer error flag:
0 : Normal return.
10 : Invalid input data (see restrictions).

Restrictions:
mx >= 1, my >= 1, lwrk >= mx*(kx+1)+my*(ky+1), kwrk >= mx+my
tx(kx+1) <= x(i-1) <= x(i) <= tx(nx-kx), i=2,...,mx
ty(ky+1) <= y(j-1) <= y(j) <= ty(ny-ky), j=2,...,my

Other Subroutines Required:
pda_fpbisp, pda_fpbspl

References:
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Latest Update: march 1987
PDA_BSPDOC
Documentation for working with piecewise polynomial functions in 
B-representation.

Origin:
SLATEC / CAMSUN
SUBROUTINE PDA_BSPDOC

***BEGIN PROLOGUE PDA_BSPDOC
***PURPOSE Documentation for BSPLINE, a package of subprograms for working with piecewise polynomial functions in B-representation.
***LIBRARY SLATEC
***CATEGORY E, E1A, K, Z
***TYPE ALL (PDA_BSPDOC-A)
***KEYWORDS B-SPLINE, DOCUMENTATION, SPLINES
***AUTHOR Amos, D. E., (SNLA)
***DESCRIPTION

Abstract

PDA_BSPDOC is a non-executable, B-spline documentary routine. The narrative describes a B-spline and the routines necessary to manipulate B-splines at a fairly high level. The basic package described herein is that of reference 5 with names altered to prevent duplication and conflicts with routines from reference 3. The call lists used here are also different. Work vectors were added to ensure portability and proper execution in an overlay environment. These work arrays can be used for other purposes except as noted in BSPVN. While most of the original routines in reference 5 were restricted to orders 20 or less, this restriction was removed from all routines except the quadrature routine BSQAD. (See the section below on differentiation and integration for details.)

The subroutines referenced below are single precision routines. Corresponding double precision versions are also part of the package, and these are referenced by prefixing a D in front of the single precision name. For example, BVALU and PDA_DBVALU are the single and double precision versions for evaluating a B-spline or any of its derivatives in the B-representation.

****Description of B-Splines****

A collection of polynomials of fixed degree K-1 defined on a subdivision \((X(I),X(I+1)), I=1,...,M-1\) of \((A,B)\) with \(X(1)=A\), \(X(M)=B\) is called a B-spline of order \(K\). If the spline has \(K-2\) continuous derivatives on \((A,B)\), then the B-spline is simply called a spline of order \(K\). Each of the \(M-1\) polynomial pieces has \(K\) coefficients, making a total of \(K(M-1)\) parameters. This B-spline and its derivatives have \(M-2\) jumps at the subdivision points \(X(I), I=2,...,M-1\). Continuity requirements at these subdivision points add constraints and reduce the number of free parameters. If a B-spline is continuous at each of the \(M-2\) subdivision points, there are \(K(M-1)-(M-2)\) free parameters; if in addition the B-spline has continuous first derivatives, there are \(K(M-1)-2(M-2)\) free parameters, etc., until we get to a spline where we have \(K(M-1)-(K-1)(M-2) = M+K-2\) free parameters.
Thus, the principle is that increasing the continuity of derivatives decreases the number of free parameters and conversely.

The points at which the polynomials are tied together by the continuity conditions are called knots. If two knots are allowed to come together at some \( X(I) \), then we say that we have a knot of multiplicity 2 there, and the knot values are the \( X(I) \) value. If we reverse the procedure of the first paragraph, we find that adding a knot to increase multiplicity increases the number of free parameters and, according to the principle above, we thereby introduce a discontinuity in what was the highest continuous derivative at that knot. Thus, the number of free parameters is \( N = NU + K - 2 \) where \( NU \) is the sum of multiplicities at the \( X(I) \) values with \( X(1) \) and \( X(M) \) of multiplicity 1 (\( NU = M \) if all knots are simple, i.e., for a spline, all knots have multiplicity 1.) Each knot can have a multiplicity of at most \( K \). A B-spline is commonly written in the B-representation

\[
Y(X) = \sum (A(I) \times B(I,X), I=1, N)
\]

to show the explicit dependence of the spline on the free parameters or coefficients \( A(I) \times BCOEF(I) \) and basis functions \( B(I,X) \). These basis functions are themselves special B-splines which are zero except on (at most) \( K \) adjoining intervals where each \( B(I,X) \) is positive and, in most cases, hat or bell-shaped. In order for the nonzero part of \( B(I,X) \) to be a spline covering \( (X(1),X(2)) \), it is necessary to put \( K-1 \) knots to the left of \( A \) and similarly for \( B(N,X) \) to the right of \( B \). Thus, the total number of knots for this representation is \( NU + 2K - 2 = N + K \). These knots are carried in an array \( T(*) \) dimensioned by at least \( N+K \). From the construction, \( A=T(K) \) and \( B=T(N+1) \) and the spline is defined on \( T(K) \leq X \leq T(N+1) \). The nonzero part of each basis function lies in the interval \( (T(I),T(I+K)) \). In many problems where extrapolation beyond A or B is not anticipated, it is common practice to set \( T(1)=T(2)=\ldots=T(K)=A \) and \( T(N+1)=T(N+2)=\ldots=T(N+K)=B \). In summary, since \( T(K) \) and \( T(N+1) \) as well as interior knots can have multiplicity \( K \), the number of free parameters is \( N = \text{sum of multiplicities} - K \). The fact that each \( B(I,X) \) function is nonzero over at most \( K \) intervals means that for a given \( X \) value, there are at most \( K \) nonzero terms of the sum. This leads to banded matrices in linear algebra problems, and references 3 and 6 take advantage of this in constructing higher level routines to achieve speed and avoid ill-conditioning.

****Basic Routines****

The basic routines which most casual users will need are those concerned with direct evaluation of splines or B-splines. Since the B-representation, denoted by \( (T,BCOEF,N,K) \), is preferred because of numerical stability, the knots \( T(*) \), the B-spline coefficients \( BCOEF(*) \), the number of coefficients \( N \),
and the order \( K \) of the polynomial pieces (of degree \( K-1 \)) are usually given. While the knot array runs from \( T(1) \) to \( T(N+K) \), the B-spline is normally defined on the interval \( T(K) \leq X \leq T(N+1) \). To evaluate the B-spline or any of its derivatives on this interval, one can use

\[
Y = \text{BVALU}(T, \text{BCOEF}, N, K, ID, X, \text{INBV}, \text{WORK})
\]

where \( ID \) is an integer for the \( ID \)-th derivative, \( 0 \leq ID \leq K-1 \). \( ID=0 \) gives the zero-th derivative or B-spline value at \( X \).

If \( X < T(K) \) or \( X > T(N+1) \), whether by mistake or the result of round off accumulation in incrementing \( X \), \text{BVALU} \) gives a diagnostic. \( \text{INBV} \) is an initialization parameter which is set to 1 on the first call. Distinct splines require distinct \( \text{INBV} \) parameters. \( \text{WORK} \) is a scratch vector of length at least \( 3*K \).

When more conventional communication is needed for publication, physical interpretation, etc., the B-spline coefficients can be converted to piecewise polynomial (PP) coefficients. Thus, the breakpoints (distinct knots) \( XI(*) \), the number of polynomial pieces \( LXI \), and the (right) derivatives \( C(*,J) \) at each breakpoint \( XI(J) \) are needed to define the Taylor expansion to the right of \( XI(J) \) on each interval \( XI(J) \leq X < XI(J+1) \), \( J=1, LXI \) where \( XI(1)=A \) and \( XI(LXI+1)=B \).

These are obtained from the \((T, \text{BCOEF}, N, K)\) representation by

\[
\text{CALL BSPPP}(T, \text{BCOEF}, N, K, \text{LDC}, C, XI, LXI, \text{WORK})
\]

where \( \text{LDC} \geq K \) is the leading dimension of the matrix \( C \) and \( \text{WORK} \) is a scratch vector of length at least \( K*(N+3) \).

Then the PP-representation \((C, XI, LXI, K)\) of \( Y(X) \), denoted by \( Y(J,X) \) on each interval \( XI(J) \leq X < XI(J+1) \), is

\[
Y(J,X) = \sum C(I,J)*((X-XI(J))**(I-1))/\text{factorial}(I-1), \ I=1,K
\]

for \( J=1, \ldots, LXI \). One must view this conversion from the B- to the PP-representation with some skepticism because the conversion may lose significant digits when the B-spline varies in an almost discontinuous fashion. To evaluate the B-spline or any of its derivatives using the PP-representation, one uses

\[
Y = \text{PPVAL}(\text{LDC}, C, XI, LXI, K, ID, X, \text{INPPV})
\]

where \( ID \) and \( \text{INPPV} \) have the same meaning and usage as \( ID \) and \( \text{INBV} \) in \text{BVALU}.

To determine to what extent the conversion process loses digits, compute the relative error \( \text{ABS}((Y1-Y2)/Y2) \) over the \( X \) interval with \( Y1 \) from \text{PPVAL} and \( Y2 \) from \text{BVALU}. A major reason for considering \text{PPVAL} is that evaluation is much faster than that from \text{BVALU}. 

\[
\text{SUN/194.3 — User-callable routines}
\]

\[48\]

\[\text{PDA_BSPDOC}\]
Recall that when multiple knots are encountered, jump type discontinuities in the B-spline or its derivatives occur at these knots, and we need to know that BVALU and PPVAL return right limiting values at these knots except at \( X=B \) where left limiting values are returned. These values are used for the Taylor expansions about left end points of breakpoint intervals. That is, the derivatives \( C(*,J) \) are right derivatives. Note also that a computed \( X \) value which, mathematically, would be a knot value may differ from the knot by a round off error. When this happens in evaluating a discontinuous B-spline or some discontinuous derivative, the value at the knot and the value at \( X \) can be radically different. In this case, setting \( X \) to a \( T \) or \( XI \) value makes the computation precise. For left limiting values at knots other than \( X=B \), see the prologues to BVALU and other routines.

****Interpolation****

\( \text{BINTK is used to generate B-spline parameters (T,BCOEF,N,K)} \)
which will interpolate the data by calls to BVALU. A similar interpolation can also be done for cubic splines using BINT4 or the code in reference 7. If the PP-representation is given, one can evaluate this representation at an appropriate number of abscissas to create data then use BINTK or BINT4 to generate the B-representation.

****Differentiation and Integration****

Derivatives of B-splines are obtained from BVALU or PPVAL. Integrals are obtained from BSQAD using the B-representation (T,BCOEF,N,K) and PPQAD using the PP-representation (C,XI,LXI,K). More complicated integrals involving the product of a of a function \( F \) and some derivative of a B-spline can be evaluated with BFQAD or PFQAD using the B- or PP-representations respectively. All quadrature routines, except for PPQAD, are limited in accuracy to 18 digits or working precision, whichever is smaller. PPQAD is limited to working precision only. If orders greater than 20 are required, use BFQAD with \( F(X) = 1. \)

****Extrapolation****

Extrapolation outside the interval \((A,B)\) can be accomplished easily by the PP-representation using PPVAL. However, caution should be exercised, especially when several knots are located at \( A \) or \( B \) or when the extrapolation is carried significantly beyond \( A \) or \( B \). On the other hand, direct evaluation with BVALU outside \( A=T(K).LE.X.LE.T(N+1)=B \) produces an error message, and some manipulation of the knots and coefficients are needed to extrapolate with BVALU. This process is described in reference 6.
****Curve Fitting and Smoothing****

Unless one has many accurate data points, direct interpolation is not recommended for summarizing data. The results are often not in accordance with intuition since the fitted curve tends to oscillate through the set of points. Monotone splines (reference 7) can help curb this undulating tendency but constrained least squares is more likely to give an acceptable fit with fewer parameters. Subroutine FC, described in reference 6, is recommended for this purpose. The output from this fitting process is the B-representation.

**** Routines in the B-Spline Package ****

Single Precision Routines

The subroutines referenced below are SINGLE PRECISION routines. Corresponding DOUBLE PRECISION versions are also part of the package and these are referenced by prefixing a D in front of the single precision name. For example, BVALU and PDA_DBVALU are the SINGLE and DOUBLE PRECISION versions for evaluating a B-spline or any of its derivatives in the B-representation.

BINT4 - interpolates with splines of order 4
BINTK - interpolates with splines of order k
BSQAD - integrates the B-representation on subintervals
PPQAD - integrates the PP-representation
BFQAD - integrates the product of a function F and any spline derivative in the B-representation
PFQAD - integrates the product of a function F and any spline derivative in the PP-representation
BVALU - evaluates the B-representation or a derivative
PPVAL - evaluates the PP-representation or a derivative
INTRV - gets the largest index of the knot to the left of x
BSPPP - converts from B- to PP-representation
BSPVD - computes nonzero basis functions and derivatives at x
BSPDR - sets up difference array for BSPEV
BSPEV - evaluates the B-representation and derivatives
BSVPN - called by BSPEV, BSPVD, BSPPP and BINTK for function and derivative evaluations

Auxiliary Routines

BSGQ8, PPGQ8, BNSLV, BNFAC, PDA_XERMSG, DBSGQ8, DPPGQ8, PDA_DNSLV, PDA_DBNFAC

Machine Dependent Routines

PDA_I1MACH, R1MACH, PDA_D1MACH

***REFERENCES***

2. D. E. Amos, Quadrature subroutines for splines and B-splines, Report SAND79-1825, Sandia Laboratories,
December 1979.

***ROUTINES CALLED (NONE)***

***REVISION HISTORY (YYMMDD)***
810223 DATE WRITTEN
861211 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900723 PURPOSE section revised. (WRB)
920501 Reformatted the REFERENCES section. (WRB)

***END PROLOGUE PDA_BSPDOC***
PDA_C2NAG

Convert FFTPACK complex Fourier transform array into equivalent NAG arrays

Description:
This subroutine re-orders and normalises the supplied array of Fourier coefficients (as produced by FFTPACK subroutine PDA_CFFTF) so that the returned arrays looks like the equivalent arrays returned by NAG routine C06FCF.

The real and imaginary coefficients produced by PDA_CFFTF are numerically larger than the corresponding C06FCF coefficients by a factor of SQRT(NP), and are stored differently. NAG uses two separate one-dimensional arrays to store the real and imaginary coefficients, whereas FFTPACK stored them in a single two dimensional array (each row holds a pair of corresponding real and imaginary coefficients).

Invocation:
CALL PDA_C2NAG( NP, R, X, Y )

Arguments:
NP = INTEGER (Given)
The number of points in the transform.

R(2, NP) = REAL (Given)
The input coefficients, in FFTPACK format.

X( NP ) = REAL (Returned)
The real coefficients, in NAG format.

Y( NP ) = REAL (Returned)
The imaginary coefficients, in NAG format.

Implementation Status:
A double precision version PDA_DC2NAG of the routine exists.
PDA_CFFTB
Unnormalized inverse of PDA_CFFTF.

**Origin:**
FFTPACK / NETLIB

**Implementation Status:**
A double precision version PDA_DCFFTB of the routine has been added.
subroutine pda_cfftb(n,c,wsave)

subroutine pda_cfftb computes the backward complex discrete fourier transform (the fourier synthesis). equivalently, pda_cfftb computes a complex periodic sequence from its fourier coefficients. the transform is defined below at output parameter c.

a call of pda_cfftf followed by a call of pda_cfftb will multiply the sequence by n.

the array wsave which is used by subroutine pda_cfftb must be initialized by calling subroutine pda_cffti(n,wsave).

input parameters

n the length of the complex sequence c. the method is more efficient when n is the product of small primes.

c a complex array of length n which contains the sequence

wsave a real work array which must be dimensioned at least 4n+15 in the program that calls pda_cfftb. the wsave array must be initialized by calling subroutine pda_cffti(n,wsave) and a different wsave array must be used for each different value of n. this initialization does not have to be repeated so long as n remains unchanged thus subsequent transforms can be obtained faster than the first. the same wsave array can be used by pda_cfftf and pda_cfftb.

output parameters

for j=1,...,n

\[ c(j) = \sum_{k=1}^{n} c(k) \cdot \exp(i*(j-1)*(k-1)*2*\pi/n) \]

where \( i = \sqrt{-1} \)

wsave contains initialization calculations which must not be destroyed between calls of subroutine pda_cfftf or pda_cfftb.
PDA_CFFTF
Forward transform of a complex periodic sequence.

Origin:
FFTPACK / NETLIB

Implementation Status:
A double precision version PDA_DCFFTF of the routine has been added.
subroutine pda_cfftf(n,c,wsave)

subroutine pda_cfftf computes the forward complex discrete fourier transform (the fourier analysis). equivalently, pda_cfftf computes the fourier coefficients of a complex periodic sequence. the transform is defined below at output parameter c.

the transform is not normalized. to obtain a normalized transform the output must be divided by n. otherwise a call of pda_cfftf followed by a call of pda_cfftb will multiply the sequence by n.

the array wsave which is used by subroutine pda_cfftf must be initialized by calling subroutine pda_cffti(n,wsave).

input parameters

n the length of the complex sequence c. the method is more efficient when n is the product of small primes. n

c a complex array of length n which contains the sequence

wsave a real work array which must be dimensioned at least 4n+15 in the program that calls pda_cfftf. the wsave array must be initialized by calling subroutine pda_cffti(n,wsave) and a different wsave array must be used for each different value of n. this initialization does not have to be repeated so long as n remains unchanged thus subsequent transforms can be obtained faster than the first. the same wsave array can be used by pda_cfftf and pda_cfftb.

output parameters

for j=1,...,n

\[ c(j) = \text{the sum from } k=1,...,n \text{ of } \]
\[ c(k) \times \exp(-i \times (j-1) \times (k-1) \times 2 \times \pi / n) \]
where \( i = \sqrt{-1} \)

wsave contains initialization calculations which must not be destroyed between calls of subroutine pda_cfftf or pda_cfftb
PDA_CFFTI
Initialize PDA_CFFTF and PDA_CFFTB.

Origin:
FFTPACK / NETLIB

Implementation Status:
A double precision version PDA_DCFFTI of the routine has been added.
subroutine pda_cffti(n,wsave)

subroutine pda_cffti initializes the array wsave which is used in both pda_cfftf and pda_cfftb. The prime factorization of n together with a tabulation of the trigonometric functions are computed and stored in wsave.

Input parameter

n the length of the sequence to be transformed

Output parameter

wsave a work array which must be dimensioned at least 4*n+15
the same work array can be used for both pda_cfftf and pda_cfftb as long as n remains unchanged. Different wsave arrays are required for different values of n. The contents of wsave must not be changed between calls of pda_cfftf or pda_cfftb.
PDA_CHE2D
Evaluates a 2-dimensional Chebyshev polynomial

Description:
This routine evaluates a two-dimensional Chebyshev polynomial for one or more arguments. It uses Clenshaw's recurrence relationship twice.

Invocation:
CALL PDA_CHE2D( NPTS, XMIN, XMAX, X, YMIN, YMAX, Y, XDEG, YDEG, MCOEF, CC, NW, WORK, EVAL, IFAIL )

Arguments:
XMIN = DOUBLE PRECISION (Given)
The lower endpoint of the range of the fit along the first dimension. The Chebyshev series representation is in terms of a normalised variable, evaluated as (2x - (XMAX + XMIN)) / (XMAX - XMIN), where x is the original variable. XMIN must be less than XMAX.

XMAX = DOUBLE PRECISION (Given)
The upper endpoint of the range of the fit along the second dimension. See XMIN.

X ( NPTS ) = DOUBLE PRECISION (Given)
The co-ordinates along the first dimension for which the Chebyshev polynomial is to be evaluated.

YMIN = DOUBLE PRECISION (Given)
The lower endpoint of the range of the fit along the first dimension. The Chebyshev series representation is in terms of a normalised variable, evaluated as (2y - (YMAX + YMIN)) / (YMAX - YMIN), where y is the original variable. YMIN must be less than YMAX.

YMAX = DOUBLE PRECISION (Given)
The upper endpoint of the range of the fit along the second dimension. See YMIN.

Y = DOUBLE PRECISION (Given)
The co-ordinate along the second dimension for which the Chebyshev polynomial is to be evaluated.

XDEG = INTEGER (Given)
The degree of the polynomial along the first dimension.

YDEG = INTEGER (Given)
The degree of the polynomial along the second dimension.

MCOEF = INTEGER (Given)
The number of coefficients. This must be at least the product of (XDEG+1) * (YDEG+1).

CC ( MCOEF ) = DOUBLE PRECISION (Given)
The Chebyshev coefficients. These should be the order such that CCij is in CC( i*(YDEG+1)+j+1 ) for i=0,XDEG; j=0,YDEG. In other words the opposite order to Fortran standard.

NW = INTEGER (Given)
The number of elements in the work array. It must be at least XDEG + 1.

WORK(NW) = DOUBLE PRECISION (Returned)
Workspace.

EVAL ( NPTS ) = DOUBLE PRECISION (Returned)
The evaluated polynomial for the supplied arguments. Should an element of argument X lie beyond the range [XMIN,XMAX], IFAIL=7 is returned.
IFAIL = INTEGER (Returned)
The status. A value of 0 indicates that the routine completed successfully. Positive values indicate
the following errors:
IFAIL = 1 XMAX less than or equal to XMIN IFAIL = 2 YMAX less than or equal to YMIN IFAIL = 3 NCOEF less than 1. IFAIL = 4 XDEG or YDEG less than 1. IFAIL = 5 Number of coefficients is too
great, namely (XDEG+1)*(YDEG+1) is greater than NCOEF. IFAIL = 6 Y lies outside the range
YMIN to YMAX. IFAIL = 7 An element of X lies outside the range XMIN to XMAX.

Notes:

- A single precision version of this function is available, named PDA_CHE2R.
PDA_COVMAT

Approximates the covariance matrix of normal order statistics

Description:
This routine computes and normalises the David-Johnson approximation for the covariance matrix of normal order statistics. The value V11 can be calculated using the PDA_V11 routine and the values of EX1, EX2 and SUMM2 using PDA_NSCOR.

Invocation:
CALL PDA_COVMAT( V, N, MDIM, V11, EX1, EX2, SUMM2, IFAULT )

Arguments:
V( MDIM, N ) = DOUBLE PRECISION (Returned)
The covariance approximation.

N = INTEGER (Given)
The sample size.

MDIM = INTEGER (Given)
First dimension of V as declared in the calling routine.

V11 = DOUBLE PRECISION (Given)
Exact value of the extreme variance V(1,1).

EX1 = DOUBLE PRECISION (Given)
Absolute expected value of the smallest order statistic from a size N sample.

EX2 = DOUBLE PRECISION (Given)
Absolute expected value of the second smallest order statistic from a size N sample.

SUMM2 = DOUBLE PRECISION (Given)
Sum of squares of expected values order statistics for a sample of size N.

IFAULT = INTEGER (Returned)
Failure indicator. Zero for success, otherwise N is out of bounds.

Origin:
Applied Statistics / Statlib Archive

Copyright:
The Royal Statistical Society.
PDA_CURFIT
Smooth spline approximation. Knots can be given or determined by the routine.

Origin:
DIERCKX / NETLIB
subroutine pda_curfit(iopt,m,x,y,w,xb,xe,k,s,nest,n,t,c,fp, * wrk,lwrk,iwrk,ier)

given the set of data points \((x(i),y(i))\) and the set of positive
numbers \(w(i),i=1,2,...,m\), subroutine pda_curfit determines a smooth spline
approximation of degree \(k\) on the interval \(xb \leq x \leq xe\).
if \(iopt=-1\) pda_curfit calculates the weighted least-squares spline
according to a given set of knots.
if \(iopt\geq0\) the number of knots of the spline \(s(x)\) and the position
\(t(j),j=1,2,...,n\) is chosen automatically by the routine. the smooth-
ness of \(s(x)\) is then achieved by minimalizing the discontinuity
jumps of the \(k\)-th derivative of \(s(x)\) at the knots \(t(j),j=k+2,k+3,...,
\(n-k-1\). the amount of smoothness is determined by the condition that
\(f(p)=\sum(w(i)\cdot(y(i)-s(x(i))))^2\) be \(\leq s\), with \(s\) a given non-
negative constant, called the smoothing factor.
the fit \(s(x)\) is given in the b-spline representation (b-spline coef-
ficients \(c(j),j=1,2,...,n-k-1\) and can be evaluated by means of
subroutine pda_splev.
calling sequence:
   call pda_curfit(iopt,m,x,y,w,xb,xe,k,s,nest,n,t,c,fp,wrk, * lwrk,iwrk,ier)
parameters:
iopt : integer flag. on entry iopt must specify whether a weighted
least-squares spline (iopt=-1) or a smoothing spline (iopt=
0 or 1) must be determined. if iopt=0 the routine will start
with an initial set of knots \(t(i)=xb\), \(t(i+k+1)=xe\), \(i=1,2,...
k+1\). if iopt=1 the routine will continue with the knots
found at the last call of the routine.
attention: a call with iopt=1 must always be immediately
preceded by another call with iopt=1 or iopt=0.
unchanged on exit.
m : integer. on entry m must specify the number of data points.
m > k. unchanged on exit.
x : real array of dimension at least (m). before entry, \(x(i)\)
must be set to the \(i\)-th value of the independent variable \(x\),
for \(i=1,2,...,m\). these values must be supplied in strictly
ascending order. unchanged on exit.
y : real array of dimension at least (m). before entry, \(y(i)\)
must be set to the \(i\)-th value of the dependent variable \(y\),
for \(i=1,2,...,m\). unchanged on exit.
w : real array of dimension at least (m). before entry, \(w(i)\)
must be set to the \(i\)-th value in the set of weights. the
\(w(i)\) must be strictly positive. unchanged on exit.
see also further comments.
xb,xe : real values. on entry xb and xe must specify the boundaries
of the approximation interval. \(xb=x(1), xe=x(m)\).
unchanged on exit.
k : integer. on entry k must specify the degree of the spline.
1\(\leq k\leq 5\). it is recommended to use cubic splines (\(k=3\)).
the user is strongly dissuaded from choosing \(k\) even, together
with a small \(s\)-value. unchanged on exit.
s : real. On entry (in case iopt>=0) s must specify the smoothing factor. s >=0. Unchanged on exit.
for advice on the choice of s see further comments.

nest : integer. On entry nest must contain an over-estimate of the total number of knots of the spline returned, to indicate the storage space available to the routine. nest >=2*k+2.
in most practical situations nest=m/2 will be sufficient.
always large enough is nest=m+k+1, the number of knots needed for interpolation (s=0). Unchanged on exit.
n : integer.
unless ier =10 (in case iopt >=0), n will contain the total number of knots of the spline approximation returned.
if the computation mode iopt=1 is used this value of n should be left unchanged between subsequent calls.
in case iopt=-1, the value of n must be specified on entry.
t : real array of dimension at least (nest).
on successful exit, this array will contain the knots of the spline, i.e. the position of the interior knots t(k+2), t(k+3) ... , t(n-k-1) as well as the position of the additional knots t(1)=t(2)=...=t(k+1)=xb and t(n-k)=...=t(n)=xe needed for the b-spline representation.
if the computation mode iopt=1 is used, the values of t(1), t(2), ..., t(n) should be left unchanged between subsequent calls. if the computation mode iopt=-1 is used, the values t(k+2), ..., t(n-k-1) must be supplied by the user, before entry. see also the restrictions (ier=10).
c : real array of dimension at least (nest).
on successful exit, this array will contain the coefficients c(1), c(2), ..., c(n-k-1) in the b-spline representation of s(x)
fp : real. Unless ier=10, fp contains the weighted sum of squared residuals of the spline approximation returned.

wrk : real array of dimension at least (m*(k+1)+nest*(7+3*k)). used as working space. if the computation mode iopt=1 is used, the values wrk(1), ..., wrk(n) should be left unchanged between subsequent calls.
lwrk : integer. On entry, lwrk must specify the actual dimension of the array wrk as declared in the calling (sub)program. lwrk must not be too small (see wrk). Unchanged on exit.
iwrk : integer array of dimension at least (nest). used as working space. if the computation mode iopt=1 is used, the values iwrk(1), ..., iwrk(n) should be left unchanged between subsequent calls.

ier : integer. Unless the routine detects an error, ier contains a non-positive value on exit, i.e.
ier=0 : normal return. the spline returned has a residual sum of squares fp such that abs(fp-s)/s <= tol with tol a relative tolerance set to 0.001 by the program.
ier=-1 : normal return. the spline returned is an interpolating spline (fp=0).
ier=-2 : normal return. the spline returned is the weighted least-squares polynomial of degree k. in this extreme case fp gives the upper bound fp0 for the smoothing factor s.
ier=1 : error. the required storage space exceeds the available storage space, as specified by the parameter nest.
probably causes: nest too small. If nest is already large (say nest > m/2), it may also indicate that s is too small.
The approximation returned is the weighted least-squares spline according to the knots \( t(1), t(2), \ldots, t(n) \). (n=nest)
The parameter fp gives the corresponding weighted sum of squared residuals (fp>s).

ier=2: error. A theoretically impossible result was found during the iteration process for finding a smoothing spline with fp = s. Probably causes: s too small.
There is an approximation returned but the corresponding weighted sum of squared residuals does not satisfy the condition \( \frac{|fp-s|}{s} < \text{tol} \).

ier=3: error. The maximal number of iterations maxit (set to 20 by the program) allowed for finding a smoothing spline with fp=s has been reached. Probably causes: s too small.
There is an approximation returned but the corresponding weighted sum of squared residuals does not satisfy the condition \( \frac{|fp-s|}{s} < \text{tol} \).

ier=10: error. On entry, the input data are controlled on validity. The following restrictions must be satisfied.
-1 < iopt < 1, 1 ≤ k ≤ 5, m > k, nest > 2k+2, w(i) > 0, i = 1, 2, ..., m
\( x_b <= x(1) < x(2) < ... < x(m) <= x_e \)
\( lwrk >= (k+1)*m + nest*(7+3*k) \)

if iopt = -1:
\( 2k+2 <= n <= \min(nest, m+k+1) \)
\( x_b < t(k+2) < t(k+3) < ... < t(n-k-1) < x_e \)
The Schoenberg-Whitney conditions, i.e. there must be a subset of data points \( x_x(j) \) such that \( t(j) < x_x(j) < t(j+k+1), j = 1, 2, ..., n-k-1 \)

if iopt = 0:
\( s >= 0 \)
\( s = 0: \text{nest} >= m+k+1 \)
If one of these conditions is found to be violated, control is immediately repassed to the calling program. In that case there is no approximation returned.

Further comments:
By means of the parameter s, the user can control the tradeoff between closeness of fit and smoothness of fit of the approximation.
If s is too large, the spline will be too smooth and signal will be lost; if s is too small the spline will pick up too much noise. In the extreme cases the program will return an interpolating spline if \( s = 0 \) and the weighted least-squares polynomial of degree k if s is very large. Between these extremes, a properly chosen s will result in a good compromise between closeness of fit and smoothness of fit.
To decide whether an approximation, corresponding to a certain s is satisfactory the user is highly recommended to inspect the fits graphically.

Recommended values for s depend on the weights w(i). If these are taken as \( 1/d(i) \) with \( d(i) \) an estimate of the standard deviation of \( y(i) \), a good s-value should be found in the range \( (m-sqrt(2*m), m + sqrt(2*m)) \). If nothing is known about the statistical error in \( y(i) \) each w(i) can be set equal to one and s determined by trial and error, taking account of the comments above. The best is then to start with a very large value of s (to determine the least-squares polynomial and the corresponding upper bound \( fp_0 \) for s) and then to...
progressively decrease the value of \( s \) (say by a factor 10 in the beginning, i.e. \( s=fp0/10, fp0/100, \ldots \) and more carefully as the approximation shows more detail) to obtain closer fits.

to economize the search for a good \( s \)-value the program provides with different modes of computation. at the first call of the routine, or whenever he wants to restart with the initial set of knots the user must set \( iopt=0 \).

if \( iopt=1 \) the program will continue with the set of knots found at the last call of the routine. this will save a lot of computation time if \( pda\_curfit \) is called repeatedly for different values of \( s \). the number of knots of the spline returned and their location will depend on the value of \( s \) and on the complexity of the shape of the function underlying the data. but, if the computation mode \( iopt=1 \) is used, the knots returned may also depend on the \( s \)-values at previous calls (if these were smaller). therefore, if after a number of trials with different \( s \)-values and \( iopt=1 \), the user can finally accept a fit as satisfactory, it may be worthwhile for him to call \( pda\_curfit \) once more with the selected value for \( s \) but now with \( iopt=0 \). indeed, \( pda\_curfit \) may then return an approximation of the same quality of fit but with fewer knots and therefore better if data reduction is also an important objective for the user.

other subroutines required:
- \( pda\_fpback \), \( pda\_fpbspl \), \( pda\_fpchec \), \( pda\_fpcurf \), \( pda\_fpdisc \),
- \( pda\_fpgivs \), \( pda\_fpknot \), \( pda\_fprati \), \( pda\_fprota \)

references:

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creation date: may 1979
latest update: march 1987
PDA_D1MACH
Floating point machine dependent constants

Origin:
SLATEC / CAMSUN

Implementation Status:
Separate versions for ‘alpha_OSF1’ and ‘sun4_Solaris’ exist.
DOUBLE PRECISION FUNCTION PDA_D1MACH (I)

***BEGIN PROLOGUE  PDA_D1MACH
***PURPOSE  Return floating point machine dependent constants.
***LIBRARY  SLATEC
***CATEGORY  R1
***TYPE  DOUBLE PRECISION (R1MACH-S, PDA_D1MACH-D)
***KEYWORDS  MACHINE CONSTANTS
***AUTHOR  Fox, P. A., (Bell Labs)
          Hall, A. D., (Bell Labs)
          Schryer, N. L., (Bell Labs)
***DESCRIPTION

PDA_D1MACH can be used to obtain machine-dependent parameters for the
local machine environment. It is a function subprogram with one
(input) argument, and can be referenced as follows:

\[ D = PDA_D1MACH(I) \]

where I=1,...,5. The (output) value of D above is determined by
the (input) value of I. The results for various values of I are
discussed below.

PDA_D1MACH(1) = B**(EMIN-1), the smallest positive magnitude.
PDA_D1MACH(2) = B**EMAX*(1 - B**(-T)), the largest magnitude.
PDA_D1MACH(3) = B**(-T), the smallest relative spacing.
PDA_D1MACH(4) = B**((1-T), the largest relative spacing.
PDA_D1MACH(5) = LOG10(B)

Assume double precision numbers are represented in the T-digit,
base-B form

\[ \text{sign} \ (B**E) \cdot (X(1)/B) + \ldots + (X(T)/B**T) \]

where 0 .LE. X(I) .LT. B for I=1,...,T, 0 .LT. X(1), and
EMIN .LE. E .LE. EMAX.

The values of B, T, EMIN and EMAX are provided in PDA_I1MACH as
follows:

\[ \begin{align*}
PDA_I1MACH(10) &= B, \text{ the base.} \\
PDA_I1MACH(14) &= T, \text{ the number of base-B digits.} \\
PDA_I1MACH(15) &= EMIN, \text{ the smallest exponent E.} \\
PDA_I1MACH(16) &= EMAX, \text{ the largest exponent E.}
\end{align*} \]

To alter this function for a particular environment, the desired
set of DATA statements should be activated by removing the C from
column 1. Also, the values of PDA_D1MACH(1) - PDA_D1MACH(4) should be
checked for consistency with the local operating system.

***REFERENCES  P. A. Fox, A. D. Hall and N. L. Schryer, Framework for
a portable library, ACM Transactions on Mathematical
Software 4, 2 (June 1978), pp. 177-188.

***ROUTINES CALLED  PDA_XERMSG
***REVISION HISTORY (YYMMDD)
750101  DATE WRITTEN
890213  REVISION DATE from Version 3.2
891214  Prologue converted to Version 4.0 format. (BAB)
900315  CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
900618  Added DEC RISC constants. (WRB)
900723  Added IBM RS 6000 constants. (WRB)
900911  Added SUN 386i constants. (WRB)
910710  Added HP 730 constants. (SMR)
911114  Added Convex IEEE constants. (WRB)
920121  Added SUN -r8 compiler option constants. (WRB)
920229  Added Touchstone Delta i860 constants. (WRB)
920501  Reformatted the REFERENCES section. (WRB)
920625  Added CONVEX -p8 and -pd8 compiler option constants.
        (BKS, WRB)
930201  Added DEC Alpha and SGI constants. (RWC and WRB)
950404  If index out of range, return value zero, but return.
        (HME).

***END PROLOGUE  PDA_D1MACH
PDA_DB2INK
Determines the parameters of a 2-D, interpolation function for gridded data.

Origin:
CMLIB / CAMSUN

Implementation Status:
The warning messages are no longer printed. The same information is returned in the argument IFAIL.
SUBROUTINE PDA_DB2INK(X,NX,Y,NY,FCN,LDF,
  :    KX,KY,TX,TY,BCOEF,
  :    WORK,IFLAG,STATUS)

C***BEGIN PROLOGUE  DB2INK
C***DATE WRITTEN  25 MAY 1982
C***REVISION DATE  25 MAY 1982
C***CATEGORY NO. E1A
C***KEYWORDS  INTERPOLATION, TWO-DIMENSIONS, GRIDDED DATA, SPLINES,
C PIECEWISE POLYNOMIALS
C***AUTHOR  BOISVERT, RONALD, NBS
C SCIENTIFIC COMPUTING DIVISION
C NATIONAL BUREAU OF STANDARDS
C WASHINGTON, DC 20234
C***PURPOSE  DOUBLE PRECISION VERSION OF B2INK.
C DB2INK DETERMINES A PIECEWISE POLYNOMIAL FUNCTION THAT
C INTERPOLATES TWO-DIMENSIONAL GRIDDED DATA. USERS SPECIFY
C THE POLYNOMIAL ORDER (DEGREE+1) OF THE INTERPOLANT AND
C (OPTIONALLY) THE KNOT SEQUENCE.
C***DESCRIPTION

C DB2INK determines the parameters of a function that interpolates
C the two-dimensional gridded data \( (X(i),Y(j),FCN(i,j)) \) for \( i=1,\ldots,NX \)
C and \( j=1,\ldots,NY \). The interpolating function and its derivatives may
C subsequently be evaluated by the function DB2VAL.

C The interpolating function is a piecewise polynomial function
C represented as a tensor product of one-dimensional B-splines. The
C form of this function is

\[
S(x,y) = \sum_{i=1}^{NX} \sum_{j=1}^{NY} a_{ij} U_i(x) V_j(y)
\]

C where the functions \( U(i) \) and \( V(j) \) are one-dimensional B-spline
C basis functions. The coefficients \( a(i,j) \) are chosen so that

\[
S(X(i),Y(j)) = FCN(i,j) \quad \text{for } i=1,\ldots,NX \text{ and } j=1,\ldots,NY
\]

C Note that for each fixed value of \( y \) \( S(x,y) \) is a piecewise
C polynomial function of \( x \) alone, and for each fixed value of \( x \) \( S(x, y) \)
C is a piecewise polynomial function of \( y \) alone. In one dimension
C a piecewise polynomial may be created by partitioning a given
C interval into subintervals and defining a distinct polynomial piece
C on each one. The points where adjacent subintervals meet are called
C knots. Each of the functions \( U(i) \) and \( V(j) \) above is a piecewise
C polynomial.

C Users of DB2INK choose the order (degree+1) of the polynomial
C pieces used to define the piecewise polynomial in each of the \( x \) and
C \( y \) directions (\( KX \) and \( KY \)). Users also may define their own knot
C sequence in \( x \) and \( y \) separately (\( TX \) and \( TY \)). If \( IFLAG=0 \), however,
C DB2INK will choose sequences of knots that result in a piecewise
C polynomial interpolant with \( KX-2 \) continuous partial derivatives in
C x and KY-2 continuous partial derivatives in y. (KX knots are taken
C near each endpoint in the x direction, not-a-knot end conditions
C are used, and the remaining knots are placed at data points if KX
C is even or at midpoints between data points if KX is odd. The y
C direction is treated similarly.)
C
C After a call to DB2INK, all information necessary to define the
C interpolating function are contained in the parameters NX, NY, KX,
C KY, TX, TY, and BCOEF. These quantities should not be altered until
C after the last call of the evaluation routine DB2VAL.
C
C I N P U T
C ---------
C
C X Double precision 1D array (size NX)
C Array of x abscissae. Must be strictly increasing.
C
C NX Integer scalar (.GE. 3)
C Number of x abscissae.
C
C Y Double precision 1D array (size NY)
C Array of y abscissae. Must be strictly increasing.
C
C NY Integer scalar (.GE. 3)
C Number of y abscissae.
C
C FCN Double precision 2D array (size LDF by NY)
C Array of function values to interpolate. FCN(I,J) should
C contain the function value at the point (X(I),Y(J))
C
C LDF Integer scalar (.GE. NX)
C The actual leading dimension of FCN used in the calling
C calling program.
C
C KX Integer scalar (.GE. 2, .LT. NX)
C The order of spline pieces in x.
C (Order = polynomial degree + 1)
C
C KY Integer scalar (.GE. 2, .LT. NY)
C The order of spline pieces in y.
C (Order = polynomial degree + 1)
C
C I N P U T O R O U T P U T
C -----------------------------
C
C TX Double precision 1D array (size NX+KX)
C The knots in the x direction for the spline interpolant.
C If IFLAG=0 these are chosen by DB2INK.
C If IFLAG=1 these are specified by the user.
C (Must be non-decreasing.)
C
C TY Double precision 1D array (size NY+KY)
PDA_DB2INK

SUN/194.3 — User-callable routines

C The knots in the y direction for the spline interpolant.
C If IFLAG=0 these are chosen by DB2INK.
C If IFLAG=1 these are specified by the user.
C (Must be non-decreasing.)

C OUTPUT
----------

C BCOEF Double precision 2D array (size NX by NY)
C Array of coefficients of the B-spline interpolant.
C This may be the same array as FCN.

C MISCELLANEOUS
---------------------

C WORK Double precision 1D array (size NX*NY + max( 2*KX*(NX+1),
C 2*KY*(NY+1) ))
C Array of working storage.

C IFLAG Integer scalar.
C On input: 0 == knot sequence chosen by DB2INK
C 1 == knot sequence chosen by user.
C On output: 1 == successful execution - Starlink modification
C 2 == IFLAG out of range
C 3 == NX out of range
C 4 == KX out of range
C 5 == X not strictly increasing
C 6 == TX not non-decreasing
C 7 == NY out of range
C 8 == KY out of range
C 9 == Y not strictly increasing
C 10 == TY not non-decreasing

C STATUS Integer. Starlink error status.

C REFERENCES CARL DE BOOR, A PRACTICAL GUIDE TO SPLINES,
C CARL DE BOOR, EFFICIENT COMPUTER MANIPULATION OF TENSOR
C PRODUCTS, ACM TRANSACTIONS ON MATHEMATICAL SOFTWARE,
C VOL. 5 (1979), PP. 173-182.
C ROUTINES CALLED DBTPCF,DBKNOT
C END PROLOGUE DB2INK
PDA_DB2VAL
Evaluates the piecewise polynomial interpolating function constructed by the routine PDA_DB2INK, or one of its partial derivatives.

Origin:
CMLIB / CAMSUN

Implementation Status:
The warning messages are no longer printed. The same information is returned in the argument IFAIL.
SUBROUTINE PDA_DB2VAL(XVAL,YVAL,IDX,IDY,TX,TY, 
: NX, NY, KK, KY, BCOEF, WORK, 
: RVALUE, IFAIL, STATUS)  
C***BEGIN PROLOGUE DB2VAL
C***DATE WRITTEN  25 MAY 1982
C***REVISION DATE  25 MAY 1982
C***CATEGORY NO.  E1A
C***KEYWORDS INTERPOLATION, TWO-DIMENSIONS, GRIDDED DATA, SPLINES, 
C PIECEWISE POLYNOMIALS
C***AUTHOR BOISVERT, RONALD, NBS
C SCIENTIFIC COMPUTING DIVISION
C NATIONAL BUREAU OF STANDARDS
C WASHINGTON, DC 20234
C***PURPOSE DB2VAL EVALUATES THE PIECEWISE POLYNOMIAL INTERPOLATING 
C FUNCTION CONSTRUCTED BY THE ROUTINE DB2INK OR ONE OF ITS 
C PARTIAL DERIVATIVES.
C DOUBLE PRECISION VERSION OF B2VAL.
C***DESCRIPTION
C
C DB2VAL evaluates the tensor product piecewise polynomial 
C interpolant constructed by the routine DB2INK or one of its 
C derivatives at the point (XVAL,YVAL). To evaluate the interpolant 
C itself, set IDX=IDY=0, to evaluate the first partial with respect 
C to x, set IDX=1,IDY=0, and so on.
C
C DB2VAL returns 0.0E0 if (XVAL,YVAL) is out of range. That is, if 
C XVAL.LT.TX(1) .OR. XVAL.GT.TX(NX+KK) .OR. 
C YVAL.LT.TY(1) .OR. YVAL.GT.TY(NY+KY)
C If the knots TX and TY were chosen by DB2INK, then this is 
C equivalent to 
C XVAL.LT.X(1) .OR. XVAL.GT.X(NX)+EPSX .OR. 
C YVAL.LT.Y(1) .OR. YVAL.GT.Y(NY)+EPSY
C where EPSX = 0.1*(X(NX)-X(NX-1)) and EPSY = 0.1*(Y(NY)-Y(NY-1)).
C
C The input quantities TX, TY, NX, NY, KK, KY, and BCOEF should be 
C unchanged since the last call of DB2INK.

C
C I N P U T
C -------
C
C XVAL  Double precision scalar
C X coordinate of evaluation point.
C
C YVAL  Double precision scalar
C Y coordinate of evaluation point.
C
C IDX  Integer scalar
C X derivative of piecewise polynomial to evaluate.
C
C IDY  Integer scalar
C Y derivative of piecewise polynomial to evaluate.
C TX  Double precision 1D array (size NX*KX)
C Sequence of knots defining the piecewise polynomial in
C the x direction.  (Same as in last call to DB2INK.)
C
C TY  Double precision 1D array (size NY*KY)
C Sequence of knots defining the piecewise polynomial in
C the y direction.  (Same as in last call to DB2INK.)
C
C NX  Integer scalar
C The number of interpolation points in x.
C  (Same as in last call to DB2INK.)
C
C NY  Integer scalar
C The number of interpolation points in y.
C  (Same as in last call to DB2INK.)
C
C KX  Integer scalar
C Order of polynomial pieces in x.
C  (Same as in last call to DB2INK.)
C
C KY  Integer scalar
C Order of polynomial pieces in y.
C  (Same as in last call to DB2INK.)
C
C BCOEF Double precision 2D array (size NX by NY)
C The B-spline coefficients computed by DB2INK.
C
C WORK  Double precision 1D array (size 3*max(KX,KY) + KY)
C A working storage array.
C
C IFAIL  A returned error value.
C
C RVALUE  The interpolated value.
C
C STATUS  Integer. Starlink status report.
C
C***REFERENCES  CARL DE BOOR, A PRACTICAL GUIDE TO SPLINES,
C***ROUTINES CALLED  DINTRV,DBVAL2
C***END PROLOGUE  DB2VAL
PDA_DBESJ1
Bessel function of first kind of order one.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
DOUBLE PRECISION FUNCTION PDA_DBESJ1 (X, STATUS)

***BEGIN PROLOGUE  PDA_DBESJ1
***PURPOSE  Compute the Bessel function of the first kind of order one.
***LIBRARY  SLATEC (FNLIB)
***CATEGORY  C10A1
***TYPE  DOUBLE PRECISION (BESJ1-S, PDA_DBESJ1-D)
***KEYWORDS  BESSEL FUNCTION, FIRST KIND, FNLIB, PDA_ORDER ONE,
             SPECIAL FUNCTIONS
***AUTHOR  Fullerton, W., (LANL)
***DESCRIPTION

PDA_DBESJ1(X) calculates the double precision Bessel function of the
first kind of order one for double precision argument X.

Series for BJ1 on the interval 0. to 1.60000E+01
with weighted error 1.16E-33
log weighted error 32.93
significant figures required 32.36
decimal places required 33.57

STATUS  Returned error status.  
The status must be zero on entry.  This
routine does not check the status on entry.

***REFERENCES (NONE)
***ROUTINES CALLED  PDA_D1MACH, PDA_D9B1MP, PDA_DCSEVL, PDA_INITDS, PDA_XERMSG
***REVISION HISTORY (YYMMDD)
  780601  DATE WRITTEN
  890531  Changed all specific intrinsics to generic.  (WRB)
  890531  REVISION DATE from Version 3.2
  891214  Prologue converted to Version 4.0 format.  (BAB)
  900315  CALLs to XERROR changed to CALLs to PDA_XERMSG.  (THJ)
  910401  Corrected error in code which caused values to have the
          wrong sign for arguments less than 4.0.  (WRB)
  950404  Implement status.  (HME)
***END PROLOGUE  PDA_DBESJ1
PDA_DBINTK
Compute B-representation of an interpolating spline. Knots must be given.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DBINTK (X, Y, T, N, K, BCOEF, Q, WORK, STATUS)

***BEGIN PROLOGUE  PDA_DBINTK
***PURPOSE  Compute the B-representation of a spline which interpolates
given data.
***LIBRARY  SLATEC
***CATEGORY  E1A
***TYPE  DOUBLE PRECISION (BINTK-S, PDA_DBINTK-D)
***KEYWORDS  B-SPLINE, DATA FITTING, INTERPOLATION
***AUTHOR  Amos, D. E., (SNLA)
***DESCRIPTION

Written by Carl de Boor and modified by D. E. Amos

Abstract  **** a double precision routine ****

PDA_DBINTK is the PDA_SPLINT routine of the reference.

PDA_DBINTK produces the B-spline coefficients, BCOEF, of the
B-spline of order K with knots T(I), I=1,...,N+K, which
takes on the value Y(I) at X(I), I=1,...,N. The spline or
any of its derivatives can be evaluated by calls to PDA_DBVALU.

The I-th equation of the linear system A*BCOEF = B for the
coefficients of the interpolant enforces interpolation at
X(I), I=1,...,N. Hence, B(I) = Y(I), for all I, and A is
a band matrix with 2K-1 bands if A is invertible. The matrix
A is generated row by row and stored, diagonal by diagonal,
in the rows of Q, with the main diagonal going into row K.
The banded system is then solved by a call to PDA_DBNFAC (which
constructs the triangular factorization for A and stores it
again in Q), followed by a call to PDA_DBNSLV (which then
obtains the solution BCOEF by substitution). PDA_DBNFAC does no
pivoting, since the total positivity of the matrix A makes
this unnecessary. The linear system to be solved is
(theoretically) invertible if and only if

    T(I) .LT. X(I) .LT. T(I+K),

for all I. Equality is permitted on the left for I=1 and on the right
for I=N when K knots are used at X(1) or X(N). Otherwise,
violation of this condition is certain to lead to an error.

Description of Arguments

Input  X,Y,T are double precision
   X  - vector of length N containing data point abscissa
       in strictly increasing order.
   Y  - corresponding vector of length N containing data
       point ordinates.
   T  - knot vector of length N+K
       Since T(1),...,T(K) .LE. X(1) and T(N+1),...,T(N+K)
       .GE. X(N), this leaves only N-K knots (not nec-
       essarily X(I) values) interior to (X(1),X(N))
   N  - number of data points, N .GE. K
K - order of the spline, K .GE. 1

Output
BCOEF - a vector of length N containing the B-spline coefficients
Q - a work vector of length (2*K-1)*N, containing the triangular factorization of the coefficient matrix of the linear system being solved. The coefficients for the interpolant of an additional data set (X(I),YY(I)), I=1,...,N with the same abscissa can be obtained by loading YY into BCOEF and then executing
   CALL PDA_DBNSLV (Q,2K-1,N,K-1,K-1,BCOEF)
WORK - work vector of length 2*K
STATUS - Returned error status.

   The status must be zero on entry. This routine does not check the status on entry.

Error Conditions
   Improper input is a fatal error
   Singular system of equations is a fatal error

***REFERENCES
Carl de Boor, Package for calculating with B-splines, SIAM Journal on Numerical Analysis 14, 3 (June 1977), pp. 441-472.

***ROUTINES CALLED PDA_DBNFAC, PDA_DBNSLV, PDA_DBSPVN, PDA_XERMSG

***REVISION HISTORY (YYMMDD)
800901 DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
890831 Modified array declarations. (WRB)
890831 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
900326 Removed duplicate information from DESCRIPTION section. (WRB)
920501 Reformatted the REFERENCES section. (WRB)
950403 Implement status. (HME)

***END PROLOGUE PDA_DBINTK
PDA_DBOLS
Solve \( E \times x = f \) (in least squares sense) with bounds on \( x \). \( E \) is a matrix, \( x \) and \( f \) are vectors.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DBOLS (W, MDW, MROWS, NCOLS, BL, BU, IND, IOPT, X, 
  + RNORM, MODE, RW, IW, STATUS)

***BEGIN PROLOGUE PDA_DBOLS
***PURPOSE Solve the problem
   E*X = F (in the least squares sense)
   with bounds on selected X values.
***LIBRARY SLATEC
***CATEGORY K1A2A, G2E, G2H1, G2H2
***TYPE DOUBLE PRECISION (SBOLS-S, PDA_DBOLS-D)
***KEYWORDS BOUNDS, CONSTRAINTS, INEQUALITY, LEAST SQUARES, LINEAR
***AUTHOR Hanson, R. J., (SNLA)
***DESCRIPTION

**** All INPUT and OUTPUT real variables are DOUBLE PRECISION ****

The user must have dimension statements of the form:

   DIMENSION W(MDW,NCOLS+1), BL(NCOLS), BU(NCOLS),
   * X(NCOLS+NX), RW(5*NCOLS)
   INTEGER IND(NCOLS), IOPT(1+NI), IW(2*NCOLS)

(Here NX=number of extra locations required for option 4; NX=0
for no options; NX=NCOLS if this option is in use. Here NI=number
of extra locations required for options 1-6; NI=0 for no
options.)

INPUT
-----

W(MDW,*),MROWS,NCOLS

The array W(*,*) contains the matrix [E:F] on entry. The matrix
[E:F] has MROWS rows and NCOLS+1 columns. This data is placed in
the array W(*,*) with E occupying the first NCOLS columns and the
right side vector F in column NCOLS+1. The row dimension, MDW, of
the array W(*,*) must satisfy the inequality MDW .ge. MROWS.
Other values of MDW are errors. The values of MROWS and NCOLS
must be positive. Other values are errors. There is an exception
to this when using option 1 for accumulation of blocks of
equations. In that case MROWS is an OUTPUT variable ONLY, and the
matrix data for [E:F] is placed in W(*,*), one block of rows at a
time. MROWS contains the number of rows in the matrix after
triangularizing several blocks of equations. This is an OUTPUT
parameter ONLY when option 1 is used. See IOPT(*) CONTENTS
for details about option 1.

BL(*),BU(*),IND(*)

These arrays contain the information about the bounds that the
solution values are to satisfy. The value of IND(J) tells the
type of bound and BL(J) and BU(J) give the explicit values for the respective upper and lower bounds.

1. For IND(J)=1, require $X(J) \geq BL(J)$.
   (the value of BU(J) is not used.)
2. For IND(J)=2, require $X(J) \leq BU(J)$.
   (the value of BL(J) is not used.)
3. For IND(J)=3, require $X(J) \geq BL(J)$ and $X(J) \leq BU(J)$.
4. For IND(J)=4, no bounds on $X(J)$ are required.
   (the values of BL(J) and BU(J) are not used.)

Values other than 1,2,3 or 4 for IND(J) are errors. In the case IND(J)=3 (upper and lower bounds) the condition $BL(J) > BU(J)$ is an error.

---------
IOPT(*)
---------
This is the array where the user can specify nonstandard options for PDA_DBOLSM( ). Most of the time this feature can be ignored by setting the input value IOPT(1)=99. Occasionally users may have needs that require use of the following subprogram options. For details about how to use the options see below: IOPT(*) CONTENTS.

<table>
<thead>
<tr>
<th>Option Number</th>
<th>Brief Statement of Purpose</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Return to user for accumulation of blocks of least squares equations.</td>
</tr>
<tr>
<td>2</td>
<td>Check lengths of all arrays used in the subprogram.</td>
</tr>
<tr>
<td>3</td>
<td>Standard scaling of the data matrix, E.</td>
</tr>
<tr>
<td>4</td>
<td>User provides column scaling for matrix E.</td>
</tr>
<tr>
<td>5</td>
<td>Provide option array to the low-level subprogram PDA_DBOLSM( ).</td>
</tr>
<tr>
<td>6</td>
<td>Move the IOPT(*) processing pointer.</td>
</tr>
<tr>
<td>99</td>
<td>No more options to change.</td>
</tr>
</tbody>
</table>

---------
X(*)
---------
This array is used to pass data associated with option 4. Ignore this parameter if this option is not used. Otherwise see below: IOPT(*) CONTENTS.

OUTPUT
--------

X(*),RNORM
--------
The array X(*) contains a solution (if MODE .ge.0 or .eq.-22) for the constrained least squares problem. The value RNORM is the minimum residual vector length.
---

**MODE**
---

The sign of MODE determines whether the subprogram has completed normally, or encountered an error condition or abnormal status. A value of MODE \(\geq 0\) signifies that the subprogram has completed normally. The value of MODE (\(\geq 0\)) is the number of variables in an active status: not at a bound nor at the value ZERO, for the case of free variables. A negative value of MODE will be one of the cases -37, -36, ..., -22, or -17, ..., -2. Values \(< -1\) correspond to an abnormal completion of the subprogram. To understand the abnormal completion codes see below: ERROR MESSAGES for PDA_DBOLS(). AN approximate solution will be returned to the user only when max. iterations is reached, MODE=-22. Values for MODE=-37, ..., -22 come from the low-level subprogram PDA_DBOLSM(). See the section ERROR MESSAGES for PDA_DBOLSM() in the documentation for PDA_DBOLSM().

---

**STATUS**
---

Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

-------

**RW(*),IW(*)**
-------

These are working arrays with 5*NCOLS and 2*NCOLS entries. (normally the user can ignore the contents of these arrays, but they must be dimensioned properly.)

**IOPT(*) CONTENTS**
-------

The option array allows a user to modify internal variables in the subprogram without recompiling the source code. A central goal of the initial software design was to do a good job for most people. Thus the use of options will be restricted to a select group of users. The processing of the option array proceeds as follows: a pointer, here called LP, is initially set to the value 1. This value is updated as each option is processed. At the pointer position the option number is extracted and used for locating other information that allows for options to be changed. The portion of the array IOPT(*) that is used for each option is fixed; the user and the subprogram both know how many locations are needed for each option. A great deal of error checking is done by the subprogram on the contents of the option array. Nevertheless it is still possible to give the subprogram optional input that is meaningless. For example option 4 uses the locations \(X(NCOLS+IOFF),...,X(NCOLS+IOFF+NCOLS-1)\) for passing scaling data. The user must manage the allocation of these locations.
This option allows the user to solve problems with a large number of rows compared to the number of variables. The idea is that the subprogram returns to the user (perhaps many times) and receives new least squares equations from the calling program unit. Eventually the user signals "that's all" and then computes the solution with one final call to subprogram PDA_DBOLS(). The value of MROWS is an OUTPUT variable when this option is used. Its value is always in the range 0 .le. MROWS .le. NCOLS+1. It is equal to the number of rows after the triangularization of the entire set of equations. If LP is the processing pointer for IOPT(*), the usage for the sequential processing of blocks of equations is

IOPT(LP)=1
Move block of equations to W(*,*) starting at
the first row of W(*,*)
IOPT(LP+3)=# of rows in the block; user defined

The user now calls PDA_DBOLS( ) in a loop. The value of IOPT(LP+1) directs the user's action. The value of IOPT(LP+2) points to where the subsequent rows are to be placed in W(*,*)

.<LOOP
. CALL PDA_DBOLS()
. IF(IOPT(LP+1) .EQ. 1) THEN
. . IOPT(LP+3)=# OF ROWS IN THE NEW BLOCK; USER DEFINED
. . PLACE NEW BLOCK OF IOPT(LP+3) ROWS IN
. . W(*,*) STARTING AT ROW IOPT(LP+2).
. . IF( THIS IS THE LAST BLOCK OF EQUATIONS ) THEN
. . . IOPT(LP+1)=2
. . <--------CYCLE LOOP
. . ELSE IF (IOPT(LP+1) .EQ. 2) THEN
. <--------EXIT LOOP SOLUTION COMPUTED IF MODE .GE. 0
. . ELSE
. . ERROR CONDITION; SHOULD NOT HAPPEN.
. <END LOOP

Use of this option adds 4 to the required length of IOPT(*).

This option is useful for checking the lengths of all arrays used by PDA_DBOLS() against their actual requirements for this problem. The idea is simple: the user’s program unit passes the declared dimension information of the arrays. These values are compared against the problem-dependent needs within the subprogram. If any of the dimensions are too small an error message is printed and a negative value of MODE is returned, -11 to -17. The printed error message tells how long the dimension should be. If LP is the processing pointer for IOPT(*),
IOPT(LP)=2
IOPT(LP+1)=Row dimension of W(*,*)
IOPT(LP+2)=Col. dimension of W(*,*)
IOPT(LP+3)=Dimensions of BL(*),BU(*),IND(*)
IOPT(LP+4)=Dimension of X(*)
IOPT(LP+5)=Dimension of RW(*)
IOPT(LP+6)=Dimension of IW(*)
IOPT(LP+7)=Dimension of IOPT(*)
.
CALL PDA_DBOLS()

Use of this option adds 8 to the required length of IOPT(*).

3

This option changes the type of scaling for the data matrix E. Nominally each nonzero column of E is scaled so that the magnitude of its largest entry is equal to the value ONE. If LP is the processing pointer for IOPT(*),

IOPT(LP)=3
IOPT(LP+1)=1,2 or 3
1= Nominal scaling as noted;
2= Each nonzero column scaled to have length ONE;
3= Identity scaling; scaling effectively suppressed.
.
CALL PDA_DBOLS()

Use of this option adds 2 to the required length of IOPT(*)..

4

This option allows the user to provide arbitrary (positive) column scaling for the matrix E. If LP is the processing pointer for IOPT(*),

IOPT(LP)=4
IOPT(LP+1)=IOFF
X(NCOLS+IOFF),...,X(NCOLS+IOFF+NCOLS-1) = Positive scale factors for cols. of E.
.
CALL PDA_DBOLS()

Use of this option adds 2 to the required length of IOPT(*) and NCOLS to the required length of X(*)

5

This option allows the user to provide an option array to the low-level subprogram PDA_DBOLSM(). If LP is the processing pointer for IOPT(*),

IOPT(LP)=5
IOPT(LP+1)= Position in IOPT(*) where option array
data for PDA_DBOLSM() begins.

CALL PDA_DBOLS()

Use of this option adds 2 to the required length of IOPT(*).

6

Move the processing pointer (either forward or backward) to the location IOPT(LP+1). The processing point is moved to entry LP+2 of IOPT(*) if the option is left with -6 in IOPT(LP). For example to skip over locations 3,...,NCOLS+2 of IOPT(*),

IOPT(1)=6
IOPT(2)=NCOLS+3
(IOPT(I), I=3,...,NCOLS+2 are not defined here.)
IOPT(NCOLS+3)=99
CALL PDA_DBOLS()

CAUTION: Misuse of this option can yield some very hard-to-find bugs. Use it with care.

99

There are no more options to change.

Only option numbers -99, -6, -5,..., -1, 1, 2,..., 6, and 99 are permitted. Other values are errors. Options -99, -1,..., -6 mean that the respective options 99, 1,..., 6 are left at their default values. An example is the option to modify the (rank) tolerance:

IOPT(1)=-3 Option is recognized but not changed
IOPT(2)=2 Scale nonzero cols. to have length ONE
IOPT(3)=99

ERROR MESSAGES for PDA_DBOLS()

WARNING IN...
PDA_DBOLS(). MDW=(I1) MUST BE POSITIVE.
   IN ABOVE MESSAGE, I1= 0
ERROR NUMBER = 2
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...
PDA_DBOLS(). NCOLS=(I1) THE NO. OF VARIABLES MUST BE POSITIVE.
   IN ABOVE MESSAGE, I1= 0
ERROR NUMBER = 3
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...
PDA_DBOLS(). FOR J=(I1), IND(J)=(I2) MUST BE 1-4.
   IN ABOVE MESSAGE, I1= 1
   IN ABOVE MESSAGE, I2= 0
ERROR NUMBER = 4
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). FOR J=(I1), BOUND BL(J)=(R1) IS .GT. BU(J)=(R2).
   IN ABOVE MESSAGE, I1= 1
   IN ABOVE MESSAGE, R1= 0.
   IN ABOVE MESSAGE, R2= ABOVE MESSAGE, I1= 0
ERROR NUMBER = 6
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). ISCALE OPTION=(I1) MUST BE 1-3.
   IN ABOVE MESSAGE, I1= 0
ERROR NUMBER = 7
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). OFFSET PAST X(NCOLS) (I1) FOR USER-PROVIDED COLUMN SCALING
   MUST BE POSITIVE.
   IN ABOVE MESSAGE, I1= 0
ERROR NUMBER = 8
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). EACH PROVIDED COL. SCALE FACTOR MUST BE POSITIVE.
   COMPONENT (I1) NOW = (R1).
   IN ABOVE MESSAGE, I1= ND. .LE. MDW=(I2).
   IN ABOVE MESSAGE, I1= 1
   IN ABOVE MESSAGE, I2= 0
ERROR NUMBER = 10
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). THE ROW DIMENSION OF W(,)=(I1) MUST BE .GE. THE NUMBER OF ROWS=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 1
ERROR NUMBER = 11
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). THE COLUMN DIMENSION OF W(,)=(I1) MUST BE .GE. NCOLS+1=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 2
ERROR NUMBER = 12
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)
WARNING IN...
PDA_DBOLS(). THE DIMENSIONS OF THE ARRAYS BL(),BU(), AND IND()=(I1) MUST BE .GE. NCOLS=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 1
ERROR NUMBER = 13
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...

PDA_DBOLS(). THE DIMENSION OF X()=(I1) MUST BE .GE. THE REQD. LENGTH=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 2
ERROR NUMBER =  14
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...

PDA_DBOLS(). THE DIMENSION OF RW()=(I1) MUST BE .GE. 5*NCOLS=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 3
ERROR NUMBER =  15
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...

PDA_DBOLS(). THE DIMENSION OF IW()=(I1) MUST BE .GE. 2*NCOLS=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 2
ERROR NUMBER =  16
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

WARNING IN...

PDA_DBOLS(). THE DIMENSION OF IOPT()=(I1) MUST BE .GE. THEREQD. LEN.=(I2).
   IN ABOVE MESSAGE, I1= 0
   IN ABOVE MESSAGE, I2= 1
ERROR NUMBER =  17
(NORMALLY A RETURN TO THE USER TAKES PLACE FOLLOWING THIS MESSAGE.)

***REFERENCES R. J. Hanson, Linear least squares with bounds and
   linear constraints, Report SAND82-1517, Sandia
   Laboratories, August 1982.

***ROUTINES CALLED PDA_DBOLS, PDA_DCOPY, PDA_DNRM2, PDA_DROT, PDA_DROTG,
   PDA_IDAMAX, PDA_XERMSG

***REVISION HISTORY (YYMMDD)

  821220 DATE WRITTEN
  891006 Cosmetic changes to prologue. (WRB)
  891006 REVISION DATE from Version 3.2
  891214 Prologue converted to Version 4.0 format. (BAB)
  900510 Convert XERRWV calls to PDA_XERMSG calls. (RWC)
  920501 Reformatted the REFERENCES section. (WRB)
  950404 Implement status. (HME)

***END PROLOGUE PDA_DBOLS

SUN/194.3 —User-callable routines

PDA_DBOLS

90
PDA_DBSQAD
Integral of a B-spline using the B-representation.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DBSQAD (T, BCOEF, N, K, X1, X2, BQUAD, WORK, STATUS)

***BEGIN PROLOGUE PDA_DBSQAD
***PURPOSE Compute the integral of a K-th order B-spline using the B-representation.
***LIBRARY SLATEC
***CATEGORY H2A2A1, E3, K6
***TYPE DOUBLE PRECISION (BSQAD-S, PDA_DBSQAD-D)
***KEYWORDS INTEGRAL OF B-SPLINES, QUADRATURE
***AUTHOR Amos, D. E., (SNLA)
***DESCRIPTION

Abstract  **** a double precision routine ****

PDA_DBSQAD computes the integral on (X1,X2) of a K-th order B-spline using the B-representation (T,BCOEF,N,K). Orders K as high as 20 are permitted by applying a 2, 6, or 10 point Gauss formula on subintervals of (X1,X2) which are formed by included (distinct) knots.

If orders K greater than 20 are needed, use DBFQAD with F(X) = 1.

The maximum number of significant digits obtainable in PDA_DBSQAD is the smaller of 18 and the number of digits carried in double precision arithmetic.

Description of Arguments
Input  T,BCOEF,X1,X2 are double precision
T   - knot array of length N+K
BCOEF - B-spline coefficient array of length N
N   - length of coefficient array
K   - order of B-spline, 1 .LE. K .LE. 20
X1,X2 - end points of quadrature interval in T(K) .LE. X .LE. T(N+1)

Output  BQUAD,WORK are double precision
BQUAD - integral of the B-spline over (X1,X2)
WORK - work vector of length 3*K
STATUS - Returned error status.
   The status must be zero on entry. This routine does not check the status on entry.

Error Conditions
Improper input is a fatal error

***ROUTINES CALLED PDA_DBVALU, PDA_DINTRV, PDA_XERMSG
***REVISION HISTORY (YMMDD)
800901 DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
890531  REVISION DATE from Version 3.2
891214  Prologue converted to Version 4.0 format. (BAB)
900315  CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
900326  Removed duplicate information from DESCRIPTION section. (WRB)
920501  Reformatted the REFERENCES section. (WRB)
950403  Implement status. (HME)

***END PROLOGUE  PDA_DBSQAD

----------------------------------------------------------->
PDA_DBVALU
Evaluate a B-spline for the function value or a derivative.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
DOUBLE PRECISION FUNCTION PDA_DBVALU (T, A, N, K, IDERIV, X, INBV, + WORK, STATUS)

***BEGIN PROLOGUE  PDA_DBVALU
***PURPOSE  Evaluate the B-representation of a B-spline at X for the function value or any of its derivatives.
***LIBRARY  SLATEC
***CATEGORY  E3, K6
***TYPE  DOUBLE PRECISION (BVALU-S, PDA_DBVALU-D)
***KEYWORDS  DIFFERENTIATION OF B-SPLINE, EVALUATION OF B-SPLINE
***AUTHOR  Amos, D. E., (SNLA)
***DESCRIPTION

Written by Carl de Boor and modified by D. E. Amos

Abstract  **** a double precision routine ****
PDA_DBVALU is the BVALUE function of the reference.

PDA_DBVALU evaluates the B-representation (T,A,N,K) of a B-spline at X for the function value on IDERIV=0 or any of its derivatives on IDERIV=1,2,...,K-1. Right limiting values (right derivatives) are returned except at the right end point X=T(N+1) where left limiting values are computed. The spline is defined on T(K) .LE. X .LE. T(N+1). PDA_DBVALU returns a fatal error message when X is outside of this interval.

To compute left derivatives or left limiting values at a knot T(I), replace N by I-1 and set X=T(I), I=K+1,N+1.

PDA_DBVALU calls PDA_DINTRV

Description of Arguments

Input  T,A,X are double precision
T - knot vector of length N+K
A - B-spline coefficient vector of length N
N - number of B-spline coefficients
N = sum of knot multiplicities-K
K - order of the B-spline, K .GE. 1
IDERIV - order of the derivative, 0 .LE. IDERIV .LE. K-1
IDERIV = 0 returns the B-spline value
X - argument, T(K) .LE. X .LE. T(N+1)
INBV - an initialization parameter which must be set to 1 the first time PDA_DBVALU is called.

Output  WORK,PDA_DBVALU are double precision
INBV - INBV contains information for efficient processing after the initial call and INBV must not be changed by the user. Distinct splines require distinct INBV parameters.
WORK - work vector of length 3*K.
PDA_DBVALU - value of the IDERIV-th derivative at X
STATUS - Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

Error Conditions
An improper input is a fatal error

***REFERENCES
Carl de Boor, Package for calculating with B-splines,
SIAM Journal on Numerical Analysis 14, 3 (June 1977), pp. 441-472.

***ROUTINES CALLED
PDA_DINTRV, PDA_XERMSG

***REVISION HISTORY (YYMMDD)
800901  DATE WRITTEN
890831  Modified array declarations. (WRB)
890911  Removed unnecessary intrinsics. (WRB)
890911  REVISION DATE from Version 3.2
891214  Prologue converted to Version 4.0 format. (BAB)
900315  CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
920501  Reformatted the REFERENCES section. (WRB)
950403  Implement status. (HME)

***END PROLOGUE  PDA_DBVALU
PDA_DC2NAG
Convert FFTPACK complex Fourier transform array into equivalent NAG arrays

Description:
c.f. PDA_C2NAG.
PDA_DCFFTB
Unnormalized inverse of PDA_DCFFTF.

Description:
c.f. PDA_CFFTB.
PDA_DCFFTF
Forward transform of a complex periodic sequence.

Description:
c.f. PDA_CFFTF.
PDA_DCFFTI
Initialize PDA_DCFFFT and PDA_DCFFTB.

Description:
c.f. PDA_CFFTII.
PDA_DCOV

Calculates the covariance matrix for a nonlinear data fitting problem

Origin:
SLATEC
**SUN/194.3 — User-callable routines**

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**PDA_DCOV**

```plaintext
SUBROUTINE PDA_DCOV (FCN, IOPT, M, N, X, FVEC, R, LDR, INFO, WA1, + WA2, WA3, WA4, STATUS)

***BEGIN PROLOGUE PDA_DCOV
***PURPOSE Calculate the covariance matrix for a nonlinear data
fitting problem. It is intended to be used after a
successful return from either PDA_DNLS1 or PDA_DNLS1E.
***LIBRARY SLATEC
***CATEGORY K1B1
***TYPE DOUBLE PRECISION (SCOV-S, DCOV-D)
***KEYWORDS COVARIANCE MATRIX, NONLINEAR DATA FITTING,
NONLINEAR LEAST SQUARES
***AUTHOR Hiebert, K. L., (SNLA)
***DESCRIPTION

1. Purpose.

PDA_DCOV calculates the covariance matrix for a nonlinear data
fitting problem. It is intended to be used after a successful
return from either PDA_DNLS1 or PDA_DNLS1E. PDA_DCOV and
PDA_DNLS1 (and PDA_DNLS1E) have compatible parameters. The
required external subroutine, FCN, is the same for all three
codes, PDA_DCOV, PDA_DNLS1, and PDA_DNLS1E.

2. Subroutine and Type Statements.

```plaintext
SUBROUTINE PDA_DCOV(FCN,IOPT,M,N,X,FVEC,R,LDR,INFO, + WA1,WA2,WA3,WA4)
INTEGER IOPT,M,N,LDR,INFO
DOUBLE PRECISION X(N),FVEC(M),R(LDR,N),WA1(N),WA2(N),WA3(N),WA4(M)
EXTERNAL FCN
```plaintext

3. Parameters. All TYPE REAL parameters are DOUBLE PRECISION

FCN is the name of the user-supplied subroutine which calculates
the functions. If the user wants to supply the Jacobian
(IOPT=2 or 3), then FCN must be written to calculate the
Jacobian, as well as the functions. See the explanation
of the IOPT argument below.
If the user wants the iterates printed in PDA_DNLS1 or PDA_DNLS1E,
then FCN must do the printing. See the explanation of NPRINT
in PDA_DNLS1 or PDA_DNLS1E. FCN must be declared in an EXTERNAL
statement in the calling program and should be written as
follows.

```plaintext
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
INTEGER IFLAG,LDFJAC,M,N
DOUBLE PRECISION X(N),FVEC(M)
----------
FJAC and LDFJAC may be ignored , if IOPT=1.
DOUBLE PRECISION FJAC(LDFJAC,N) , if IOPT=2.
DOUBLE PRECISION FJAC(N) , if IOPT=3.
----------
If IFLAG=0, the values in X and FVEC are available
for printing in PDA_DNLS1 or PDA_DNLS1E.
```
IFLAG will never be zero when FCN is called by PDA_DCOV. The values of X and FVEC must not be changed.
RETURN
----------
If IFLAG=1, calculate the functions at X and return this vector in FVEC.
RETURN
----------
If IFLAG=2, calculate the full Jacobian at X and return this matrix in FJAC. Note that IFLAG will never be 2 unless IOPT=2. FVEC contains the function values at X and must not be altered. FJAC(I,J) must be set to the derivative of FVEC(I) with respect to X(J).
RETURN
----------
If IFLAG=3, calculate the LDFJAC-th row of the Jacobian and return this vector in FJAC. Note that IFLAG will never be 3 unless IOPT=3. FJAC(J) must be set to the derivative of FVEC(LDFJAC) with respect to X(J).
RETURN
----------
END

The value of IFLAG should not be changed by FCN unless the user wants to terminate execution of PDA_DCOV. In this case, set IFLAG to a negative integer.

IOPT is an input variable which specifies how the Jacobian will be calculated. If IOPT=2 or 3, then the user must supply the Jacobian, as well as the function values, through the subroutine FCN. If IOPT=2, the user supplies the full Jacobian with one call to FCN. If IOPT=3, the user supplies one row of the Jacobian with each call. (In this manner, storage can be saved because the full Jacobian is not stored.) If IOPT=1, the code will approximate the Jacobian by forward differencing.

M is a positive integer input variable set to the number of functions.

N is a positive integer input variable set to the number of variables. N must not exceed M.

X is an array of length N. On input X must contain the value at which the covariance matrix is to be evaluated. This is usually the value for X returned from a successful run of PDA_DNLS1 (or PDA_DNLS1E). The value of X will not be changed.

FVEC is an output array of length M which contains the functions evaluated at X.

R is an output array. For IOPT=1 and 2, R is an M by N array.
For IOPT=3, R is an N by N array. On output, if INFO=1, the upper N by N submatrix of R contains the covariance matrix evaluated at X.

LDR is a positive integer input variable which specifies the leading dimension of the array R. For IOPT=1 and 2, LDR must not be less than M. For IOPT=3, LDR must not be less than N.

INFO is an integer output variable. If the user has terminated execution, INFO is set to the (negative) value of IFLAG. See description of FCN. Otherwise, INFO is set as follows.

INFO = 0 Improper input parameters (M.LE.0 or N.LE.0).
INFO = 1 Successful return. The covariance matrix has been calculated and stored in the upper N by N submatrix of R.
INFO = 2 The Jacobian matrix is singular for the input value of X. The covariance matrix cannot be calculated. The upper N by N submatrix of R contains the QR factorization of the Jacobian (probably not of interest to the user).

WA1,WA2 are work arrays of length N.
and WA3
WA4 is a work array of length M.

STATUS is an INTEGER error status. Set to zero on entry. If an error has occurred and has been reported then this will be non-zero on exit.

****REFERENCES (NONE)

****ROUTINES CALLED DENORM, DFDJC3, DQRFAC, DWUPDT, XERMSG

****REVISION HISTORY (YMMDD)
810522 DATE WRITTEN
890831 Modified array declarations. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to XERMSG. (THJ)
900510 Fixed an error message. (RWC)
970224 Now called PDA_DCOV. (PWD)

***END PROLOGUE DCOV
PDA_DEFC
Fit piecewise polynomial curve represented as B-splines (weighted least squares sense).

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DEFC (NDATA, XDATA, YDATA, SDDATA, NORD, NBKPT, BKPT,
+     MDEIN, MDEOUT, COEFF, LW, W, STATUS)

***BEGIN PROLOGUE PDA_DEFC
***PURPOSE Fit a piecewise polynomial curve to discrete data.
The piecewise polynomials are represented as B-splines.
The fitting is done in a weighted least squares sense.
***LIBRARY SLATEC
***CATEGORY K1A1A1, K1A2A, L8A3
***TYPE DOUBLE PRECISION (EFC-S, PDA_DEFC-D)
***KEYWORDS B-SPLINE, CONSTRAINED LEAST SQUARES, CURVE FITTING
***AUTHOR Hanson, R. J., (SNLA)
***DESCRIPTION

This subprogram fits a piecewise polynomial curve to discrete data. The piecewise polynomials are represented as B-splines.
The fitting is done in a weighted least squares sense.

The data can be processed in groups of modest size. The size of the group is chosen by the user. This feature may be necessary for purposes of using constrained curve fitting with subprogram DFC( ) on a very large data set.

For a description of the B-splines and usage instructions to evaluate them, see


For further discussion of (constrained) curve fitting using B-splines, see


Input. All TYPE REAL variables are DOUBLE PRECISION
NDATA,XDATA(*),
YDATA(*),
SDDATA(*)

The NDATA discrete (X,Y) pairs and the Y value standard deviation or uncertainty, SD, are in the respective arrays XDATA(*), YDATA(*), and SDDATA(*). No sorting of XDATA(*) is required. Any non-negative value of NDATA is allowed. A negative value of NDATA is an error. A zero value for any entry of SDDATA(*) will weight that data point as 1. Otherwise the weight of that data point is the reciprocal of this entry.
NORD, NBKPT, BKPT(*)

The NBKPT knots of the B-spline of order NORD are in the array BKPT(*). Normally the problem data interval will be included between the limits BKPT(NORD) and BKPT(NBKPT-NORD+1). The additional end knots BKPT(I), I=1,..., NORD-1 and I=NBKPT-NORD+2,...,NBKPT, are required to compute the functions used to fit the data. No sorting of BKPT(*) is required. Internal to PDA_DEFC() the extreme end knots may be reduced and increased respectively to accommodate any data values that are exterior to the given knot values. The contents of BKPT(*) is not changed.

NORD must be in the range 1 .LE. NORD .LE. 20.
The value of NBKPT must satisfy the condition NBKPT .GE. 2*NORD.
Other values are considered errors.

(The order of the spline is one more than the degree of the piecewise polynomial defined on each interval. This is consistent with the B-spline package convention. For example, NORD=4 when we are using piecewise cubics.)

MDEIN

An integer flag, with one of two possible values (1 or 2), that directs the subprogram action with regard to new data points provided by the user.

=1 The first time that PDA_DEFC( ) has been entered. There are NDATA points to process.
=2 This is another entry to PDA_DEFC(). The subprogram PDA_DEFC( ) has been entered with MDEIN=1 exactly once before for this problem. There are NDATA new additional points to merge and process with any previous points.
(When using PDA_DEFC() with MDEIN=2 it is important that the set of knots remain fixed at the same values for all entries to PDA_DEFC( ).)

LW

The amount of working storage actually allocated for the working array W(*). This quantity is compared with the actual amount of storage needed in PDA_DEFC( ). Insufficient storage allocated for W(*) is an error. This feature was included in PDA_DEFC because misreading the storage formula for W(*) might very well lead to subtle and hard-to-find programming bugs.
The length of the array $W(*)$ must satisfy

$$LW \geq (N_{BKPT}-NORD+3)*(NORD+1)+$$

$$N_{BKPT}+1)*(NORD+1)+$$

$$2*\max(NDATA, NBKPT)+NBKPT+NORD**2$$

Output: All TYPE REAL variables are DOUBLE PRECISION

**MDEOUT**

An output flag that indicates the status of the curve fit.

-1 A usage error of PDA_DEFC( ) occurred. The offending condition is noted with the SLATEC library error processor, PDA_XERMSG( ). In case the working array $W(*)$ is not long enough, the minimal acceptable length is printed.

1 The B-spline coefficients for the fitted curve have been returned in array $COEFF(*)$.

2 Not enough data has been processed to determine the B-spline coefficients.

The user has one of two options. Continue to process more data until a unique set of coefficients is obtained, or use the subprogram DFC( ) to obtain a specific set of coefficients. The user should read the usage instructions for DFC( ) for further details if this second option is chosen.

**COEFF(*)**

If the output value of MDEOUT=1, this array contains the unknowns obtained from the least squares fitting process. These $N=NBKPT-NORD$ parameters are the B-spline coefficients. For MDEOUT=2, not enough data was processed to uniquely determine the B-spline coefficients. In this case, and also when MDEOUT=-1, all values of COEFF(*) are set to zero.

If the user is not satisfied with the fitted curve returned by PDA_DEFC( ), the constrained least squares curve fitting subprogram DFC( ) may be required. The work done within PDA_DEFC( ) to accumulate the data can be utilized by the user, if so desired. This involves saving the first $(NBKPT-NORD+3)*(NORD+1)$ entries of $W(*)$ and providing this data to DFC( ) with the "old problem" designation. The user should read the usage instructions for subprogram DFC( ) for further details.

**STATUS**

Returned error status.

The status must be zero on entry. This
routine does not check the status on entry.

Working Array.. All TYPE REAL variables are DOUBLE PRECISION

\[ W(*) \]

This array is typed DOUBLE PRECISION. Its length is specified as an input parameter in LW as noted above. The contents of \( W(*) \) must not be modified by the user between calls to PDA_DEFC() with values of MDEIN=1,2,2,.... The first (NBKPT-NORD+3)*(NORD+1) entries of \( W(*) \) are acceptable as direct input to DFC() for an "old problem" only when MDEOUT=1 or 2.

Evaluating the Fitted Curve..

To evaluate derivative number IDER at XVAL, use the function subprogram PDA_DBVALU().

\[ F = PDA\_DBVALU(BKPT,COEFF,NBKPT-NORD,NORD,IDER, XVAL,INBV,WORKB) \]

The output of this subprogram will not be defined unless an output value of MDEOUT=1 was obtained from PDA_DEFC(), XVAL is in the data interval, and IDER is nonnegative and .LT. NORD.

The first time PDA_DBVALU() is called, INBV=1 must be specified. This value of INBV is the overwritten by PDA_DBVALU(). The array WORKB(*) must be of length at least 3*NORD, and must not be the same as the \( W(*) \) array used in the call to PDA_DEFC().

PDA_DBVALU() expects the breakpoint array BKPT(*) to be sorted.

***REFERENCES R. J. Hanson, Constrained least squares curve fitting to discrete data using B-splines, a users guide, Report SAND78-1291, Sandia Laboratories, December 1978.

***ROUTINES CALLED PDA_DEFCMN

***REVISION HISTORY (YYMMDD)

800801 DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
890531 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900510 Change Prologue comments to refer to PDA_XERMSG. (RWC)
900607 Editorial changes to Prologue to make Prologues for EFC, PDA_DEFC, FC, and DFC look as much the same as possible. (RWC)
920501 Reformatted the REFERENCES section. (WRB)
950403 Implement status. (HME)

***END PROLOGUE PDA_DEFC
PDA_DEFLT
Set default parameters for use by PDA_SUMSL.

Description:
c.f. [PDA_SUMSL]
subroutine pda_deflt(alg, iv, liv, lv, v)

*** supply ***sol (version 2.3) default values to iv and v ***

*** alg = 1 means regression constants.
*** alg = 2 means general unconstrained optimization constants.
PDA_DERF
Error function erf().

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
DOUBLE PRECISION FUNCTION PDA_DERF (X, STATUS)

***BEGIN PROLOGUE  PDA_DERF
***PURPOSE  Compute the error function.
***LIBRARY  SLATEC (FNLIB)
***CATEGORY  C8A, L5A1E
***TYPE  DOUBLE PRECISION (ERF-S, PDA_DERF-D)
***KEYWORDS  ERF, ERROR FUNCTION, FNLIB, SPECIAL FUNCTIONS
***AUTHOR  Fullerton, W., (LANL)
***DESCRIPTION

PDA_DERF(X) calculates the double precision error function for double precision argument X.

Series for ERF on the interval 0. to 1.00000E+00
with weighted error 1.28E-32
log weighted error 31.89
significant figures required 31.05
decimal places required 32.55

STATUS  Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

***REFERENCES (NONE)
***ROUTINES CALLED  PDA_D1MACH, PDA_DCSEVL, PDA_DERFC, PDA_INITDS
***REVISION HISTORY (YYMMDD)
770701  DATE WRITTEN
890531  Changed all specific intrinsics to generic.  (WRB)
890531  REVISION DATE from Version 3.2
891214  Prologue converted to Version 4.0 format.  (BAB)
900727  Added EXTERNAL statement.  (WRB)
920618  Removed space from variable name.  (RWC, WRB)
950425  Implement status.  (HME)
***END PROLOGUE  PDA_DERF
PDA_DERFC
Complementary error function erfc().

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
DOUBLE PRECISION FUNCTION PDA_DERFC (X, STATUS)

***BEGIN PROLOGUE  PDA_DERFC
***PURPOSE  Compute the complementary error function.
***LIBRARY  SLATEC (FNLIB)
***CATEGORY  C8A, L5A1E
***TYPE  DOUBLE PRECISION (ERFC-S, PDA_DERFC-D)
***KEYWORDS  COMPLEMENTARY ERROR FUNCTION, ERFC, FNLIB,
SPECIAL FUNCTIONS
***AUTHOR  Fullerton, W., (LANL)
***DESCRIPTION

PDA_DERFC(X) calculates the double precision complementary error function for double precision argument X.

Series for ERF  on the interval 0. to 1.00000E+00
               with weighted Error 1.28E-32
               log weighted Error 31.89
               significant figures required 31.05
               decimal places required 32.55

Series for ERC2 on the interval 2.50000E-01 to 1.00000E+00
               with weighted Error 2.67E-32
               log weighted Error 31.57
               significant figures required 30.31
               decimal places required 32.42

Series for ERFC on the interval 0. to 2.50000E-01
               with weighted error 1.53E-31
               log weighted error 30.82
               significant figures required 29.47
               decimal places required 31.70

STATUS  Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

***REFERENCES (NONE)
***ROUTINES CALLED  PDA_D1MACH, PDA_DCSEVL, PDA_INITDS, PDA_XERMSG
***REVISION HISTORY (YYMMDD)
  770701  DATE WRITTEN
  890531  Changed all specific intrinsics to generic. (WRB)
  890531  REVISION DATE from Version 3.2
  891214  Prologue converted to Version 4.0 format. (BAB)
  900315  CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
  920618  Removed space from variable names. (RWC, WRB)
  950425  Implement status. (HME)
***END PROLOGUE  PDA_DERFC
PDA_DGEDI
Determinant and inverse of a matrix using the factors from PDA_DGEFA.

Origin:
SLATEC / CAMSUN
SUBROUTINE PDA_DGEDI (A, LDA, N, IPVT, DET, WORK, JOB)

***BEGIN PROLOGUE  PDA_DGEDI
***PURPOSE  Compute the determinant and inverse of a matrix using the
factors computed by PDA_DGECO or PDA_DGEFA.
***LIBRARY  SLATEC (LINPACK)
***CATEGORY  D3A1, D2A1
***TYPE  DOUBLE PRECISION (SGEDI-S, PDA_DGEDI-D, CGEDI-C)
***KEYWORDS  DETERMINANT, INVERSE, LINEAR ALGEBRA, LINPACK, MATRIX
***AUTHOR  Moler, C. B., (U. of New Mexico)
***DESCRIPTION

PDA_DGEDI computes the determinant and inverse of a matrix
using the factors computed by PDA_DGECO or PDA_DGEFA.

On Entry

A  DOUBLE PRECISION(LDA, N)
the output from PDA_DGECO or PDA_DGEFA.
LDA  INTEGER
the leading dimension of the array  A  .
N  INTEGER
the order of the matrix  A  .
IPVT  INTEGER(N)
the pivot vector from PDA_DGECO or PDA_DGEFA.
WORK  DOUBLE PRECISION(N)
work vector. Contents destroyed.
JOB  INTEGER
= 11 both determinant and inverse.
   = 01  inverse only.
   = 10  determinant only.

On Return

A  inverse of original matrix if requested.
   Otherwise unchanged.
DET  DOUBLE PRECISION(2)
   determinant of original matrix if requested.
   Otherwise not referenced.
   Determinant = DET(1) * 10.0**DET(2)
   with  1.0 .LE. ABS(DET(1)) .LT. 10.0
   or  DET(1) .EQ. 0.0  .

Error Condition

A division by zero will occur if the input factor contains
a zero on the diagonal and the inverse is requested.
It will not occur if the subroutines are called correctly
and if PDA_DGECO has set RCOND .GT. 0.0 or PDA_DGEEFA has set
INFO .EQ. 0 .

***REFERENCES  J. J. Dongarra, J. R. Bunch, C. B. Moler, and G. W.
***ROUTINES CALLED  PDA_DAXPY, PDA_DSCAL, PDA_DSWAP
***REVISION HISTORY (YYMMDD)
  780814  DATE WRITTEN
  890531  Changed all specific intrinsics to generic. (WRB)
  890831  Modified array declarations. (WRB)
  890831  REVISION DATE from Version 3.2
  891214  Prologue converted to Version 4.0 format. (BAB)
  900326  Removed duplicate information from DESCRIPTION section.
          (WRB)
  920501  Reformatted the REFERENCES section. (WRB)
***END PROLOGUE  PDA_DGEDI
PDA_DGEFA
Factor a matrix using Gaussian elimination. This is needed before PDA_DGEDI.

Origin:
SLATEC / CAMSUN
SUBROUTINE PDA_DGEFA (A, LDA, N, IPVT, INFO)

***BEGIN PROLOGUE  PDA_DGEFA
***PURPOSE  Factor a matrix using Gaussian elimination.
***LIBRARY  SLATEC (LINPACK)
***CATEGORY  D2A1
***TYPE  DOUBLE PRECISION (SGEFA-S, PDA_DGEFA-D, CGEFA-C)
***KEYWORDS  GENERAL MATRIX, LINEAR ALGEBRA, LINPACK,
             MATRIX FACTORIZATION
***AUTHOR  Moler, C. B., (U. of New Mexico)
***DESCRIPTION

PDA_DGEFA factors a double precision matrix by Gaussian elimination.

PDA_DGEFA is usually called by PDA_DGECO, but it can be called
directly with a saving in time if RCOND is not needed.
(Time for PDA_DGECO) = (1 + 9/N)*(Time for PDA_DGEFA) .

On Entry

A  DOUBLE PRECISION(LDA, N)
the matrix to be factored.

LDA  INTEGER
the leading dimension of the array A .

N  INTEGER
the order of the matrix A .

On Return

A  an upper triangular matrix and the multipliers
   which were used to obtain it.
   The factorization can be written A = L*U where
   L is a product of permutation and unit lower
   triangular matrices and U is upper triangular.

IPVT  INTEGER(N)
an integer vector of pivot indices.

INFO  INTEGER
= 0  normal value.
= K  if U(K,K) .EQ. 0.0 . This is not an error
   condition for this subroutine, but it does
   indicate that PDA_DGESL or PDA_DGEDI will divide by zero
   if called. Use RCOND in PDA_DGECO for a reliable
   indication of singularity.

***REFERENCES  J. J. Dongarra, J. R. Bunch, C. B. Moler, and G. W.
***ROUTINES CALLED  PDA_DAXPY, PDA_DSCAL, PDA_IDAMAX
***REVISION HISTORY (YYMMDD)
780814  DATE WRITTEN
890831 Modified array declarations. (WRB)
890831 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900326 Removed duplicate information from DESCRIPTION section. (WRB)
920501 Reformatted the REFERENCES section. (WRB)
***END PROLOGUE PDA_DGEFA
PDA_DGEFS

Solve the problem \( A \times x = b \). \( A \) is a square matrix, \( x \) and \( b \) are vectors. Factoring of \( A \) can be re-used to solve for multi-column \( X \) and \( B \).

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
Subroutine PDA_DGEFS solves a general $N \times N$ system of double precision linear equations using LINPACK subroutines PDA_DGECO and PDA_DGESL. That is, if $A$ is an $N \times N$ double precision matrix and if $X$ and $B$ are double precision $N$-vectors, then PDA_DGEFS solves the equation

$$A \cdot X = B.$$ 

The matrix $A$ is first factored into upper and lower triangular matrices $U$ and $L$ using partial pivoting. These factors and the pivoting information are used to find the solution vector $X$. An approximate condition number is calculated to provide a rough estimate of the number of digits of accuracy in the computed solution.

If the equation $A \cdot X = B$ is to be solved for more than one vector $B$, the factoring of $A$ does not need to be performed again and the option to only solve (ITASK $> 1$) will be faster for the succeeding solutions. In this case, the contents of $A$, LDA, N and IWORK must not have been altered by the user following factorization (ITASK=1). IND will not be changed by PDA_DGEFS in this case.

**Argument Description**

- **A**
  - **DOUBLE PRECISION(LDA,N)**
  - on entry, the doubly subscripted array with dimension (LDA,N) which contains the coefficient matrix.
  - on return, an upper triangular matrix $U$ and the multipliers necessary to construct a matrix $L$ so that $A = L \cdot U$.

- **LDA**
  - **INTEGER**
  - the leading dimension of the array $A$. LDA must be greater than or equal to N. (terminal error message IND=-1)

- **N**
  - **INTEGER**
  - the order of the matrix $A$. The first $N$ elements of the array $A$ are the elements of the first column of the matrix $A$. $N$ must be greater than or equal to 1. (terminal error message IND=-2)

- **V**
  - **DOUBLE PRECISION(N)**
  - on entry, the singly subscripted array(vector) of di-
mension N which contains the right hand side B of a system of simultaneous linear equations A*X=B. on return, V contains the solution vector, X.

**ITASK INTEGER**
- If ITASK=1, the matrix A is factored and then the linear equation is solved.
- If ITASK .GT. 1, the equation is solved using the existing factored matrix A and IWORK.
- If ITASK .LT. 1, then terminal error message IND=-3 is printed.

**IND INTEGER**
- GT. 0 IND is a rough estimate of the number of digits of accuracy in the solution, X.
- LT. 0 see error message corresponding to IND below.

**WORK DOUBLE PRECISION(N)**
a singly subscripted array of dimension at least N.

**IWORK INTEGER(N)**
a singly subscripted array of dimension at least N.

**STATUS INTEGER**
Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

Error Messages Printed ***

- IND=-1 terminal N is greater than LDA.
- IND=-2 terminal N is less than 1.
- IND=-3 terminal ITASK is less than 1.
- IND=-4 terminal The matrix A is computationally singular. A solution has not been computed.
- IND=-10 warning The solution has no apparent significance.
  The solution may be inaccurate or the matrix A may be poorly scaled.

**Note**—The above terminal(*fatal*) error messages are designed to be handled by PDA_XERMSG in which LEVEL=1 (recoverable) and IFLAG=2. LEVEL=0 for warning error messages from PDA_XERMSG. Unless the user provides otherwise, an error message will be printed followed by an abort.


***ROUTINES CALLED*** PDA_D1MACH, PDA_DGECO, PDA_DGESL, PDA_XERMSG

***REVISION HISTORY (YYMMDD)***
- 800326 DATE WRITTEN
- 890531 Changed all specific intrinsics to generic. (WRB)
- 890831 Modified array declarations. (WRB)
- 890831 REVISION DATE from Version 3.2
- 891214 Prologue converted to Version 4.0 format. (BAB)
- 900315 CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
- 900510 Convert XERRWV calls to PDA_XERMSG calls. (RWC)
- 920501 Reformatted the REFERENCES section. (WRB)
- 950404 Implement status. (HME)
***END PROLOGUE PDA_DGEFS
PDA_DNAG2C
Convert NAG complex Fourier transform array into array usable by
FFTPACK routine PDA_DCFFTB

Description:
c.f. [PDA_NAG2C]
PDA_DNAG2R
Convert NAG Hermitian Fourier transform array into array usable by FFTPACK routine PDA_DRFFTB

Description:
c.f. [PDA_NAG2R].
**PDA_DNFFTB**

Backward FFT of N-dimensional complex array

**Description:**

c.f. [PDA_NFFTB]
PDA_DNFFTF
Forward FFT of N-dimensional complex array

Description:
c.f. PDA_NFFTF
PDA_DNLS1
Minimises the sum of squares of M non-linear functions

Origin:
SLATEC
SUBROUTINE PDA_DNLS1 (FCN, IOPT, M, N, X, FVEC, FJAC, LDFJAC,
+ FTOL, XTOL, GTOL, MAXFEV, EPSFCN, DIAG,
+ MODE, FACTOR, NPRINT, INFO, NFEV, NJEV,
+ IPVT, QTF, WA1, WA2, WA3, WA4, STATUS)

***BEGIN PROLOGUE DNLS1
***PURPOSE Minimize the sum of the squares of M nonlinear functions
in N variables by a modification of the Levenberg-Marquardt
algorithm.
***LIBRARY SLATEC
***CATEGORY K1B1A1, K1B1A2
***TYPE DOUBLE PRECISION (SNLS1-S, DNLS1-D)
***KEYWORDS LEVENBERG-MARQUARDT, NONLINEAR DATA FITTING,
NONLINEAR LEAST SQUARES
***AUTHOR Hiebert, K. L., (SNLA)
***DESCRIPTION

1. Purpose.

The purpose of DNLS1 is to minimize the sum of the squares of M
nonlinear functions in N variables by a modification of the
Levenberg-Marquardt algorithm. The user must provide a subrou-
tine which calculates the functions. The user has the option
of how the Jacobian will be supplied. The user can supply the
full Jacobian, or the rows of the Jacobian (to avoid storing
the full Jacobian), or let the code approximate the Jacobian by
forward-differencing. This code is the combination of the
MINPACK codes (Argonne) LMDER, LMDIF, and LMSTR.

2. Subroutine and Type Statements.

SUBROUTINE DNLS1(FCN,IOPT,M,N,X,FVEC,FJAC,LDFJAC,FTOL,XTOL,
+ GTOL,MAXFEV,EPSFCN,DIAG,MODE,FACTOR,NPRINT,INFO,
+ NFEV,NJEV,IPVT,QTF,WA1,WA2,WA3,WA4)
INTEGER IOPT,M,N,LDFJAC,MAXFEV,MODE,NPRINT,INFO
INTEGER IPVT(N)
DOUBLE PRECISION FTOL,XTOL,GTOL,M.MAXFEV,EPSFCN,DIAG,MODE,FACTOR,NPRINT,INFO,NFEV,NJEV
INTEGER IPVT(N)
DOUBLE PRECISION FTOL,XTOL,GTOL,EPSFCN,FACTOR
DOUBLE PRECISION X(N),FVEC(N),FJAC(LDFJAC,N),DIAG(N),QTF(N),
+ WA1(N),WA2(N),WA3(N),WA4(M)

3. Parameters.

Parameters designated as input parameters must be specified on
entry to DNLS1 and are not changed on exit, while parameters
designated as output parameters need not be specified on entry
and are set to appropriate values on exit from DNLS1.

FCN is the name of the user-supplied subroutine which calculate
the functions. If the user wants to supply the Jacobian
(IOPT=2 or 3), then FCN must be written to calculate the
Jacobian, as well as the functions. See the explanation
of the IOPT argument below.
If the user wants the iterates printed (NPRINT positive), then
FCN must do the printing. See the explanation of NPRINT below. FCN must be declared in an EXTERNAL statement in the calling program and should be written as follows.

```fortran
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
INTEGER IFLAG,LDFJAC,M,N
DOUBLE PRECISION X(N),FVEC(M)
----------
FJAC and LDFJAC may be ignored , if IOPT=1.
DOUBLE PRECISION FJAC(LDFJAC,N) , if IOPT=2.
DOUBLE PRECISION FJAC(N) , if IOPT=3.
----------
If IFLAG=0, the values in X and FVEC are available for printing. See the explanation of NPRINT below. IFLAG will never be zero unless NPRINT is positive. The values of X and FVEC must not be changed.
RETURN
----------
If IFLAG=1, calculate the functions at X and return this vector in FVEC.
RETURN
----------
If IFLAG=2, calculate the full Jacobian at X and return this matrix in FJAC. Note that IFLAG will never be 2 unless IOPT=2. FVEC contains the function values at X and must not be altered. FJAC(I,J) must be set to the derivative of FVEC(I) with respect to X(J).
RETURN
----------
If IFLAG=3, calculate the LDFJAC-th row of the Jacobian and return this vector in FJAC. Note that IFLAG will never be 3 unless IOPT=3. FVEC contains the function values at X and must not be altered. FJAC(J) must be set to the derivative of FVEC(LDFJAC) with respect to X(J).
RETURN
----------
END
```

The value of IFLAG should not be changed by FCN unless the user wants to terminate execution of DNLS1. In this case, set IFLAG to a negative integer.

IOPT is an input variable which specifies how the Jacobian will be calculated. If IOPT=2 or 3, then the user must supply the Jacobian, as well as the function values, through the subroutine FCN. If IOPT=2, the user supplies the full Jacobian with one call to FCN. If IOPT=3, the user supplies one row of the Jacobian with each call. (In this manner, storage can be saved because the full Jacobian is not stored.) If IOPT=1, the code will approximate the Jacobian by forward differencing.
M is a positive integer input variable set to the number of functions.

N is a positive integer input variable set to the number of variables. N must not exceed M.

X is an array of length N. On input, X must contain an initial estimate of the solution vector. On output, X contains the final estimate of the solution vector.

FVEC is an output array of length M which contains the functions evaluated at the output X.

FJAC is an output array. For IOPT=1 and 2, FJAC is an M by N array. For IOPT=3, FJAC is an N by N array. The upper N by N submatrix of FJAC contains an upper triangular matrix R with diagonal elements of non-increasing magnitude such that

\[ T \quad T \quad T \]
\[ P \cdot (JAC \cdot JAC) \cdot P = R \cdot R, \]

where P is a permutation matrix and JAC is the final calculated Jacobian. Column J of P is column IPVT(J) (see below) of the identity matrix. The lower part of FJAC contains information generated during the computation of R.

LDFJAC is a positive integer input variable which specifies the leading dimension of the array FJAC. For IOPT=1 and 2, LDFJAC must not be less than M. For IOPT=3, LDFJAC must not be less than N.

FTOL is a non-negative input variable. Termination occurs when both the actual and predicted relative reductions in the sum of squares are at most FTOL. Therefore, FTOL measures the relative error desired in the sum of squares. Section 4 contains more details about FTOL.

XTOL is a non-negative input variable. Termination occurs when the relative error between two consecutive iterates is at most XTOL. Therefore, XTOL measures the relative error desired in the approximate solution. Section 4 contains more details about XTOL.

GTOL is a non-negative input variable. Termination occurs when the cosine of the angle between FVEC and any column of the Jacobian is at most GTOL in absolute value. Therefore, GTOL measures the orthogonality desired between the function vector and the columns of the Jacobian. Section 4 contains more details about GTOL.

MAXFEV is a positive integer input variable. Termination occurs when the number of calls to FCN to evaluate the functions has reached MAXFEV.
EPSFCN is an input variable used in determining a suitable step for the forward-difference approximation. This approximation assumes that the relative errors in the functions are of the order of EPSFCN. If EPSFCN is less than the machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision. If IOPT=2 or 3, then EPSFCN can be ignored (treat it as a dummy argument).

DIAG is an array of length N. If MODE = 1 (see below), DIAG is internally set. If MODE = 2, DIAG must contain positive entries that serve as implicit (multiplicative) scale factors for the variables.

MODE is an integer input variable. If MODE = 1, the variables will be scaled internally. If MODE = 2, the scaling is specified by the input DIAG. Other values of MODE are equivalent to MODE = 1.

FACTOR is a positive input variable used in determining the initial step bound. This bound is set to the product of FACTOR and the Euclidean norm of DIAG*X if nonzero, or else to FACTOR itself. In most cases FACTOR should lie in the interval (.1,100.). 100. is a generally recommended value.

NPRINT is an integer input variable that enables controlled printing of iterates if it is positive. In this case, FCN is called with IFLAG = 0 at the beginning of the first iteration and every NPRINT iterations thereafter and immediately prior to return, with X and FVEC available for printing. Appropriate print statements must be added to FCN (see example) and FVEC should not be altered. If NPRINT is not positive, no special calls to FCN with IFLAG = 0 are made.

INFO is an integer output variable. If the user has terminated execution, INFO is set to the (negative) value of IFLAG. See description of FCN and JAC. Otherwise, INFO is set as follows

INFO = 0 improper input parameters.
INFO = 1 both actual and predicted relative reductions in the sum of squares are at most FTOL.
INFO = 2 relative error between two consecutive iterates is at most XTOL.
INFO = 3 conditions for INFO = 1 and INFO = 2 both hold.
INFO = 4 the cosine of the angle between FVEC and any column of the Jacobian is at most GTOL in absolute value.
INFO = 5 number of calls to FCN for function evaluation has reached MAXFEV.
INFO = 6  FTOL is too small. No further reduction in the sum of squares is possible.

INFO = 7  XTOL is too small. No further improvement in the approximate solution X is possible.

INFO = 8  GTOL is too small. FVEC is orthogonal to the columns of the Jacobian to machine precision.

Sections 4 and 5 contain more details about INFO.

NFEV is an integer output variable set to the number of calls to FCN for function evaluation.

NJEV is an integer output variable set to the number of evaluations of the full Jacobian. If IOPT=2, only one call to FCN is required for each evaluation of the full Jacobian. If IOPT=3, the M calls to FCN are required. If IOPT=1, then NJEV is set to zero.

IPVT is an integer output array of length N. IPVT defines a permutation matrix P such that JAC*P = Q*R, where JAC is the final calculated Jacobian, Q is orthogonal (not stored), and R is upper triangular with diagonal elements of non-increasing magnitude. Column J of P is column IPVT(J) of the identity matrix.

QTF is an output array of length N which contains the first N elements of the vector (Q transpose)*FVEC.

WA1, WA2, and WA3 are work arrays of length N.

WA4 is a work array of length M.

STATUS is an INTEGER error status. Set to zero on entry. If an error has occurred and has been reported then this will be non-zero on exit.

4. Successful Completion.

The accuracy of DNLS1 is controlled by the convergence parameters FTOL, XTOL, and GTOL. These parameters are used in tests which make three types of comparisons between the approximation X and a solution XSOL. DNLS1 terminates when any of the tests is satisfied. If any of the convergence parameters is less than the machine precision (as defined by the function R1MACH(4)), then DNLS1 only attempts to satisfy the test defined by the machine precision. Further progress is not usually possible.

The tests assume that the functions are reasonably well behaved, and, if the Jacobian is supplied by the user, that the functions and the Jacobian are coded consistently. If these conditions are not satisfied, then DNLS1 may incorrectly indicate convergence. If the Jacobian is coded correctly or IOPT=1,
then the validity of the answer can be checked, for example, by rerunning DNLS1 with tighter tolerances.

First Convergence Test. If ENORM(Z) denotes the Euclidean norm of a vector Z, then this test attempts to guarantee that

$$ENORM(FVEC) \leq (1+FTOL) \cdot ENORM(FVECS),$$

where FVECS denotes the functions evaluated at XSOL. If this condition is satisfied with $FTOL = 10^{-K}$, then the final residual norm $ENORM(FVEC)$ has $K$ significant decimal digits and INFO is set to 1 (or to 3 if the second test is also satisfied). Unless high precision solutions are required, the recommended value for $FTOL$ is the square root of the machine precision.

Second Convergence Test. If D is the diagonal matrix whose entries are defined by the array DIAG, then this test attempts to guarantee that

$$ENORM(D \cdot (X-XSOL)) \leq XTOL \cdot ENORM(D \cdot XSOL).$$

If this condition is satisfied with $XTOL = 10^{-K}$, then the larger components of $D \cdot X$ have $K$ significant decimal digits and INFO is set to 2 (or to 3 if the first test is also satisfied). There is a danger that the smaller components of $D \cdot X$ may have large relative errors, but if $MODE = 1$, then the accuracy of the components of $X$ is usually related to their sensitivity. Unless high precision solutions are required, the recommended value for $XTOL$ is the square root of the machine precision.

Third Convergence Test. This test is satisfied when the cosine of the angle between FVEC and any column of the Jacobian at $X$ is at most $GTOL$ in absolute value. There is no clear relationship between this test and the accuracy of DNLS1, and furthermore, the test is equally well satisfied at other critical points, namely maximizers and saddle points. Therefore, termination caused by this test (INFO = 4) should be examined carefully. The recommended value for $GTOL$ is zero.

5. Unsuccessful Completion.

Unsuccessful completion of DNLS1 can be due to improper input parameters, arithmetic interrupts, or an excessive number of function evaluations.

Improper Input Parameters. INFO is set to 0 if IOPT .LT. 1 or IOPT .GT. 3, or $N \leq 0$, or $M \lt N$, or for IOPT=1 or 2 $LDFJAC \leq M$, or for IOPT=3 $LDFJAC \leq N$, or $FTOL \leq 0.E0$, or $XTOL \leq 0.E0$, or $GTOL \leq 0.E0$, or MAXFEV .LE. 0, or FACTOR .LE. 0.E0.
Arithmetic Interrupts. If these interrupts occur in the FCN subroutine during an early stage of the computation, they may be caused by an unacceptable choice of X by DNLS1. In this case, it may be possible to remedy the situation by rerunning DNLS1 with a smaller value of FACTOR.

Excessive Number of Function Evaluations. A reasonable value for MAXFEV is $100*(N+1)$ for IOPT=2 or 3 and $200*(N+1)$ for IOPT=1. If the number of calls to FCN reaches MAXFEV, then this indicates that the routine is converging very slowly as measured by the progress of FVEC, and INFO is set to 5. In this case, it may be helpful to restart DNLS1 with MODE set to 1.


DNLS1 is a modification of the Levenberg-Marquardt algorithm. Two of its main characteristics involve the proper use of implicitly scaled variables (if MODE = 1) and an optimal choice for the correction. The use of implicitly scaled variables achieves scale invariance of DNLS1 and limits the size of the correction in any direction where the functions are changing rapidly. The optimal choice of the correction guarantees (under reasonable conditions) global convergence from starting points far from the solution and a fast rate of convergence for problems with small residuals.

Timing. The time required by DNLS1 to solve a given problem depends on M and N, the behavior of the functions, the accuracy requested, and the starting point. The number of arithmetic operations needed by DNLS1 is about $N^3$ to process each evaluation of the functions (call to FCN) and to process each evaluation of the Jacobian it takes $M^*N^2$ for IOPT=2 (one call to FCN), $M^*N^2$ for IOPT=1 (N calls to FCN) and $1.5M^*N^2$ for IOPT=3 (M calls to FCN). Unless FCN can be evaluated quickly, the timing of DNLS1 will be strongly influenced by the time spent in FCN.

Storage. DNLS1 requires $(M*N + 2*M + 6*N)$ for IOPT=1 or 2 and $(N^*2 + 2*M + 6*N)$ for IOPT=3 single precision storage locations and N integer storage locations, in addition to the storage required by the program. There are no internally declared storage arrays.

*Long Description:

7. Example.

The problem is to determine the values of $X(1)$, $X(2)$, and $X(3)$ which provide the best fit (in the least squares sense) of

$$X(1) + U(I)/(V(I)*X(2) + W(I)*X(3)), \quad I = 1, 15$$
to the data

\[ Y = (0.14, 0.18, 0.22, 0.25, 0.29, 0.32, 0.35, 0.39, \\
0.37, 0.58, 0.73, 0.96, 1.34, 2.10, 4.39), \]

where \( U(I) = I, \ V(I) = 16 - I, \) and \( W(I) = \min(U(I), V(I)) \). The
I-th component of \( FVEC \) is thus defined by

\[ Y(I) - (X(1) + \frac{U(I)}{V(I) \times X(2) + W(I) \times X(3)}). \]

**********

PROGRAM TEST

C

C Driver for DNLS1 example.
C

INTEGER J, IOPT, M, N, LDFJAC, MAXFEV, MODE, NPRINT, INFO, NFEV, NJEV,
* NWRITE
INTEGER IPVT(3)
DOUBLE PRECISION FTOL, XTOL, GTOL, FACTOR, FNORM, EPSFCN
DOUBLE PRECISION X(3), FVEC(15), FJAC(15,3), DIAG(3), QTQ(3),
* WA1(3), WA2(3), WA3(3), WA4(15)
DOUBLE PRECISION DENORM, DIMACH
EXTERNAL FCN
DATA NWRITE /6/

C

IOPT = 1
M = 15
N = 3

C

The following starting values provide a rough fit.

C

X(1) = 1.E0
X(2) = 1.E0
X(3) = 1.E0

C

LDFJAC = 15

C

Set FTOL and XTOL to the square root of the machine precision
and GTOL to zero. Unless high precision solutions are
required, these are the recommended settings.

C

FTOL = SQRT(R1MACH(4))
XTOL = SQRT(R1MACH(4))
GTOL = 0.E0

C

MAXFEV = 400
EPSFCN = 0.0
MODE = 1
FACTOR = 1.E2
NPRINT = 0

C

CALL DNLS1(FCN, IOPT, M, N, X, FVEC, FJAC, LDFJAC, FTOL, XTOL,
*         GTOL, MAXFEV, EPSFCN, DIAG, MODE, FACTOR, NPRINT,
* INFO,NFEV,NJEV,IPVT,QTF,WA1,WA2,WA3,WA4)
FNorm = ENORM(M,FVEC)
WRITE (NWRITE,1000) FNorm,NFEV,NJEV,INFO,(X(J),J=1,N)
STOP
1000 FORMAT (5X,'FINAL L2 NORM OF THE RESIDUALS',E15.7 //
* 5X,'NUMBER OF FUNCTION EVALUATIONS',I10 //
* 5X,'NUMBER OF JACOBIAN EVALUATIONS',I10 //
* 5X,'EXIT PARAMETER',16X,I10 //
* 5X,'FINAL APPROXIMATE SOLUTION' // 5X,3E15.7)
END
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,DUM,IDUM)
C This is the form of the FCN routine if IOPT=1,
C that is, if the user does not calculate the Jacobian.
INTEGER I,M,N,IFLAG
DOUBLE PRECISION X(N),FVEC(M),Y(15)
DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8),
* Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15)
* /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1,
* 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/
C
IF (IFLAG .NE. 0) GO TO 5
C
C Insert print statements here when NPRINT is positive.
C
RETURN
5 CONTINUE
DO 10 I = 1, M
  TMP1 = I
  TMP2 = 16 - I
  TMP3 = TMP1
  IF (I .GT. 8) TMP3 = TMP2
  FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
10 CONTINUE
RETURN
END

Results obtained with different compilers or machines may be slightly different.

FINAL L2 NORM OF THE RESIDUALS 0.9063596E-01
NUMBER OF FUNCTION EVALUATIONS 25
NUMBER OF JACOBIAN EVALUATIONS 0
EXIT PARAMETER 1
FINAL APPROXIMATE SOLUTION

0.8241058E-01 0.1133037E+01 0.2343695E+01
For IOPT=2, FCN would be modified as follows to also calculate the full Jacobian when IFLAG=2.

```fortran
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
  INTEGER I,LDFJAC,M,N,IFLAG
  DOUBLE PRECISION X(N),FVEC(M),FJAC(LDFJAC,N),Y(15)
  DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
  DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8),
     * Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15)
     * /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1,
     * 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/
  IF (IFLAG .NE. 0) GO TO 5
  C Insert print statements here when NPRINT is positive.
  RETURN
  5 CONTINUE
  IF(IFLAG.NE.1) GO TO 20
  DO 10 I = 1, M
    TMP1 = I
    TMP2 = 16 - I
    TMP3 = TMP1
    IF (I .GT. 8) TMP3 = TMP2
    FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
  10 CONTINUE
  RETURN
  C Below, calculate the full Jacobian.
  C
  20 CONTINUE
  DO 30 I = 1, M
    TMP1 = I
    TMP2 = 16 - I
    TMP3 = TMP1
    IF (I .GT. 8) TMP3 = TMP2
    TMP4 = (X(2)*TMP2 + X(3)*TMP3)**2
    FJAC(I,1) = -1.E0
    FJAC(I,2) = TMP1*TMP2/TMP4
    FJAC(I,3) = TMP1*TMP3/TMP4
  30 CONTINUE
  RETURN
END
```

For IOPT = 3, FJAC would be dimensioned as FJAC(3,3), LDFJAC would be set to 3, and FCN would be written as follows to calculate a row of the Jacobian when IFLAG=3.
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
C This is the form of the FCN routine if IOPT=3,
C that is, if the user calculates the Jacobian row by row.
INTEGER I,M,N,IFLAG
DOUBLE PRECISION X(N),FVEC(M),FJAC(N),Y(15)
DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8),
* Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15)
* /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1,
* 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/
C
IF (IFLAG .NE. 0) GO TO 5
C
C Insert print statements here when NPRINT is positive.
C
RETURN
5 CONTINUE
IF( IFLAG.NE.1) GO TO 20
DO 10 I = 1, M
   TMP1 = I
   TMP2 = 16 - I
   TMP3 = TMP1
   IF (I .GT. 8) TMP3 = TMP2
   FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
10 CONTINUE
RETURN
C
C Below, calculate the LDFJAC-th row of the Jacobian.
C
20 CONTINUE
I = LDFJAC
   TMP1 = I
   TMP2 = 16 - I
   TMP3 = TMP1
   IF (I .GT. 8) TMP3 = TMP2
   TMP4 = (X(2)*TMP2 + X(3)*TMP3)**2
   FJAC(1) = -1.E0
   FJAC(2) = TMP1*TMP2/TMP4
   FJAC(3) = TMP1*TMP3/TMP4
RETURN
END

***REFERENCES Jorge J. More, The Levenberg-Marquardt algorithm:
   implementation and theory. In Numerical Analysis
   Proceedings (Dundee, June 28 - July 1, 1977, G. A.
   Watson, Editor), Lecture Notes in Mathematics 630,

***ROUTINES CALLED DIMACH, DCKDER, DENORM, DFDJC3, DMPAR, DQRFAC,
   DWUPDT, XERMSG

***REVISION HISTORY (YYMMDD)
800301 DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
890831 Modified array declarations. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to XERMSG. (THJ)
900510 Convert XERRWV calls to XERMSG calls. (RWC)
920205 Corrected XERN1 declaration. (WRB)
920501 Reformatted the REFERENCES section. (WRB)
960916 Renamed PDA_DNLS1 and added STATUS argument. (PWD)

***END PROLOGUE DNLS1
PDA_DNLS1E
Minimises the sum of squares of M non-linear functions (easy version)

Origin:
SLATEC
**SUBROUTINE PDA_DNLS1E (FCN, IOPT, M, N, X, FVEC, TOL, NPRINT, +
INFO, IW, WA, LWA, STATUS)**

***BEGIN PROLOGUE DNLS1E***

***PURPOSE*** An easy-to-use code which minimizes the sum of the squares of M nonlinear functions in N variables by a modification of the Levenberg-Marquardt algorithm.

***LIBRARY*** SLATEC

***CATEGORY*** K1B1A1, K1B1A2

***TYPE*** DOUBLE PRECISION (SNLS1E-S, DNLS1E-D)

***KEYWORDS*** EASY-TO-USE, LEVENBERG-MARQUARDT, NONLINEAR DATA FITTING, NONLINEAR LEAST SQUARES

***AUTHOR*** Hiebert, K. L., (SNLA)

***DESCRIPTION***

1. Purpose.

The purpose of DNLS1E is to minimize the sum of the squares of M nonlinear functions in N variables by a modification of the Levenberg-Marquardt algorithm. This is done by using the more general least-squares solver DNLS1. The user must provide a subroutine which calculates the functions. The user has the option of how the Jacobian will be supplied. The user can supply the full Jacobian, or the rows of the Jacobian (to avoid storing the full Jacobian), or let the code approximate the Jacobian by forward-differencing. This code is the combination of the MINPACK codes (Argonne) LMDER1, LMDIF1, and LMSTR1.

2. Subroutine and Type Statements.

```
SUBROUTINE DNLS1E(FCN,IOPT,M,N,X,FVEC,TOL,NPRINT,
   * INTEGER IOPT,M,N,NPRINT,INFO,LWAC,IW(N)
   DOUBLE PRECISION TOL,X(N),FVEC(M),WA(LWA)
EXTERNAL FCN
```

3. Parameters. ALL TYPE REAL parameters are DOUBLE PRECISION

Parameters designated as input parameters must be specified on entry to DNLS1E and are not changed on exit, while parameters designated as output parameters need not be specified on entry and are set to appropriate values on exit from DNLS1E.

FCN is the name of the user-supplied subroutine which calculates the functions. If the user wants to supply the Jacobian (IOPT=2 or 3), then FCN must be written to calculate the Jacobian, as well as the functions. See the explanation of the IOPT argument below.

If the user wants the iterates printed (NPRINT positive), then FCN must do the printing. See the explanation of NPRINT below. FCN must be declared in an EXTERNAL statement in the calling program and should be written as follows.
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
INTEGER IFLAG,LDFJAC,M,N
DOUBLE PRECISION X(N),FVEC(M)
----------
FJAC and LDFJAC may be ignored , if IOPT=1.
DOUBLE PRECISION FJAC(LDFJAC,N) , if IOPT=2.
DOUBLE PRECISION FJAC(N) , if IOPT=3.
----------
If IFLAG=0, the values in X and FVEC are available
for printing. See the explanation of NPRINT below.
IFLAG will never be zero unless NPRINT is positive.
The values of X and FVEC must not be changed.
RETURN
----------
If IFLAG=1, calculate the functions at X and return
this vector in FVEC.
RETURN
----------
If IFLAG=2, calculate the full Jacobian at X and return
this matrix in FJAC. Note that IFLAG will never be 2 unless
IOPT=2. FVEC contains the function values at X and must
not be altered. FJAC(I,J) must be set to the derivative
of FVEC(I) with respect to X(J).
RETURN
----------
If IFLAG=3, calculate the LDFJAC-th row of the Jacobian
and return this vector in FJAC. Note that IFLAG will
never be 3 unless IOPT=3. FVEC contains the function
values at X and must not be altered. FJAC(J) must be
set to the derivative of FVEC(LDFJAC) with respect to X(J).
RETURN
----------
END

The value of IFLAG should not be changed by FCN unless the
user wants to terminate execution of DNLS1E. In this case,
set IFLAG to a negative integer.

IOPT is an input variable which specifies how the Jacobian will
be calculated. If IOPT=2 or 3, then the user must supply the
Jacobian, as well as the function values, through the
subroutine FCN. If IOPT=2, the user supplies the full
Jacobian with one call to FCN. If IOPT=3, the user supplies
one row of the Jacobian with each call. (In this manner,
storage can be saved because the full Jacobian is not stored.)
If IOPT=1, the code will approximate the Jacobian by forward
differencing.

M is a positive integer input variable set to the number of
functions.
N is a positive integer input variable set to the number of variables. N must not exceed M.

X is an array of length N. On input, X must contain an initial estimate of the solution vector. On output, X contains the final estimate of the solution vector.

FVEC is an output array of length M which contains the functions evaluated at the output X.

TOL is a non-negative input variable. Termination occurs when the algorithm estimates either that the relative error in the sum of squares is at most TOL or that the relative error between X and the solution is at most TOL. Section 4 contains more details about TOL.

NPRINT is an integer input variable that enables controlled printing of iterates if it is positive. In this case, FCN is called with IFLAG = 0 at the beginning of the first iteration and every NPRINT iterations thereafter and immediately prior to return, with X and FVEC available for printing. Appropriate print statements must be added to FCN (see example) and FVEC should not be altered. If NPRINT is not positive, no special calls of FCN with IFLAG = 0 are made.

INFO is an integer output variable. If the user has terminated execution, INFO is set to the (negative) value of IFLAG. See description of FCN and JAC. Otherwise, INFO is set as follows.

INFO = 0 improper input parameters.
INFO = 1 algorithm estimates that the relative error in the sum of squares is at most TOL.
INFO = 2 algorithm estimates that the relative error between X and the solution is at most TOL.
INFO = 3 conditions for INFO = 1 and INFO = 2 both hold.
INFO = 4 FVEC is orthogonal to the columns of the Jacobian to machine precision.
INFO = 5 number of calls to FCN has reached 100*(N+1) for IOPT=2 or 3 or 200*(N+1) for IOPT=1.
INFO = 6 TOL is too small. No further reduction in the sum of squares is possible.
INFO = 7 TOL is too small. No further improvement in the approximate solution X is possible.

Sections 4 and 5 contain more details about INFO.

IW is an INTEGER work array of length N.
WA is a work array of length LWA.

LWA is a positive integer input variable not less than

$N \times (M+5) + M$ for IOPT=1 and 2 or

$N \times (N+5) + M$ for IOPT=3.

STATUS is an INTEGER error status. Set to zero on entry.
If an error has occurred and has been reported then
this will be non-zero on exit.

4. Successful Completion.

The accuracy of DNLS1E is controlled by the convergence parameter TOL. This parameter is used in tests which make three types of comparisons between the approximation $X$ and a solution XSOL. DNLS1E terminates when any of the tests is satisfied. If TOL is less than the machine precision (as defined by the function R1MACH(4)), then DNLS1E only attempts to satisfy the test defined by the machine precision. Further progress is not usually possible. Unless high precision solutions are required, the recommended value for TOL is the square root of the machine precision.

The tests assume that the functions are reasonably well behaved, and, if the Jacobian is supplied by the user, that the functions and the Jacobian are coded consistently. If these conditions are not satisfied, then DNLS1E may incorrectly indicate convergence. If the Jacobian is coded correctly or IOPT=1, then the validity of the answer can be checked, for example, by rerunning DNLS1E with tighter tolerances.

First Convergence Test. If $\text{ENORM}(Z)$ denotes the Euclidean norm of a vector $Z$, then this test attempts to guarantee that

$$\text{ENORM}(\text{FVEC}) \leq (1 + \text{TOL}) \times \text{ENORM}(\text{FVECS}),$$

where FVECS denotes the functions evaluated at XSOL. If this condition is satisfied with TOL = $10^{-K}$, then the final residual norm $\text{ENORM}(\text{FVEC})$ has $K$ significant decimal digits and INFO is set to 1 (or to 3 if the second test is also satisfied).

Second Convergence Test. If $D$ is a diagonal matrix (implicitly generated by DNLS1E) whose entries contain scale factors for the variables, then this test attempts to guarantee that

$$\text{ENORM}(D \times (X - XSOL)) \leq \text{TOL} \times \text{ENORM}(D \times XSOL).$$

If this condition is satisfied with TOL = $10^{-K}$, then the larger components of $D \times X$ have $K$ significant decimal digits and INFO is set to 2 (or to 3 if the first test is also satisfied). There is a danger that the smaller components of $D \times X$ may have large relative errors, but the choice of $D$ is such
that the accuracy of the components of $X$ is usually related to their sensitivity.

Third Convergence Test. This test is satisfied when $\text{FVEC}$ is orthogonal to the columns of the Jacobian to machine precision. There is no clear relationship between this test and the accuracy of $\text{DNLS1E}$, and furthermore, the test is equally well satisfied at other critical points, namely maximizers and saddle points. Therefore, termination caused by this test ($\text{INFO} = 4$) should be examined carefully.

5. Unsuccessful Completion.

Unsuccessful termination of $\text{DNLS1E}$ can be due to improper input parameters, arithmetic interrupts, or an excessive number of function evaluations.

Improper Input Parameters. $\text{INFO}$ is set to 0 if $\text{IOPT} .LT. 1$ or $\text{IOPT} .GT. 3$, or $N .LE. 0$, or $M .LT. N$, or $\text{TOL} .LT. 0.E0$, or for $\text{IOPT}=1$ or 2 $LWA .LT. N*(M+5)+M$, or for $\text{IOPT}=3$ $LWA .LT. N*(N+5)+M$.

Arithmetic Interrupts. If these interrupts occur in the FCN subroutine during an early stage of the computation, they may be caused by an unacceptable choice of $X$ by $\text{DNLS1E}$. In this case, it may be possible to remedy the situation by not evaluating the functions here, but instead setting the components of $\text{FVEC}$ to numbers that exceed those in the initial $\text{FVEC}$.

Excessive Number of Function Evaluations. If the number of calls to FCN reaches $100*(N+1)$ for $\text{IOPT}=2$ or 3 or $200*(N+1)$ for $\text{IOPT}=1$, then this indicates that the routine is converging very slowly as measured by the progress of $\text{FVEC}$, and $\text{INFO}$ is set to 5. In this case, it may be helpful to restart $\text{DNLS1E}$, thereby forcing it to disregard old (and possibly harmful) information.


$\text{DNLS1E}$ is a modification of the Levenberg-Marquardt algorithm. Two of its main characteristics involve the proper use of implicitly scaled variables and an optimal choice for the correction. The use of implicitly scaled variables achieves scale invariance of $\text{DNLS1E}$ and limits the size of the correction in any direction where the functions are changing rapidly. The optimal choice of the correction guarantees (under reasonable conditions) global convergence from starting points far from the solution and a fast rate of convergence for problems with small residuals.

Timing. The time required by $\text{DNLS1E}$ to solve a given problem depends on $M$ and $N$, the behavior of the functions, the accu-
racy requested, and the starting point. The number of arithmetic operations needed by DNLS1E is about \(N^3\) to process each evaluation of the functions (call to FCN) and to process each evaluation of the Jacobian DNLS1E takes \(M \times N^2\) for \(IOPT=2\) (one call to JAC), \(M \times N^2\) for \(IOPT=1\) (\(N\) calls to FCN) and \(1.5 \times M \times N^2\) for \(IOPT=3\) (\(M\) calls to FCN). Unless FCN can be evaluated quickly, the timing of DNLS1E will be strongly influenced by the time spent in FCN.

Storage. DNLS1E requires \((M \times N + 2 \times M + 6 \times N)\) for \(IOPT=1\) or 2 and \((N^2 + 2 \times M + 6 \times N)\) for \(IOPT=3\) single precision storage locations and \(N\) integer storage locations, in addition to the storage required by the program. There are no internally declared storage arrays.

**Long Description:**

7. Example.

The problem is to determine the values of \(X(1)\), \(X(2)\), and \(X(3)\) which provide the best fit (in the least squares sense) of

\[ X(1) + \frac{U(I)}{V(I) \times X(2) + W(I) \times X(3)}, \quad I = 1, 15 \]

to the data

\[ Y = (0.14,0.18,0.22,0.25,0.29,0.32,0.35,0.39, \]
\[ 0.37,0.58,0.73,0.96,1.34,2.10,4.39), \]

where \(U(I) = I\), \(V(I) = 16-I\), and \(W(I) = \text{MIN}(U(I),V(I))\). The \(I\)-th component of \(FVEC\) is thus defined by

\[ Y(I) - (X(1) + \frac{U(I)}{V(I) \times X(2) + W(I) \times X(3)}). \]

**********

PROGRAM TEST

C
C Driver for DNLS1E example.
C
INTEGER I,IOPT,M,N,NPRINT,JNFO,LWA,NWRITE
INTEGER IW(3)
DOUBLE PRECISION TOL,FNORM,X(3),FVEC(15),WA(75)
DOUBLE PRECISION DENORM,D1MACH
EXTERNAL FCN
DATA NWRITE /6/
C
IOPT = 1
M = 15
N = 3
C
The following starting values provide a rough fit.
C
X(1) = 1.E0
X(2) = 1.E0
X(3) = 1.E0

C
LWA = 75
NPRINT = 0
C
Set TOL to the square root of the machine precision.
C
Unless high precision solutions are required,
C
this is the recommended setting.
C
TOL = SQRT(R1MACH(4))
C
CALL DNLS1E(FCN,IOPT,M,N,X,FVEC,TOL,NPRINT,
* INFO,IW,WA,LWA)
FNORM = ENORM(M,FVEC)
WRITE (NWRITE,1000) FNORM,INFO,(X(J),J=1,N)
STOP
1000 FORMAT (5X,' FINAL L2 NORM OF THE RESIDUALS','E15.7 //
* 5X,' EXIT
* 5X,' FINAL APPROXIMATE SOLUTION' // 5X,3E15.7)
END
SUBROUTINE FCN(IFLAG,M,N,X,FVEC,DUM,IDUM)
C
This is the form of the FCN routine if IOPT=1,
C
that is, if the user does not calculate the Jacobian.
INTEGER I,M,N,IFLAG
DOUBLE PRECISION X(N),FVEC(M),Y(15)
DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8),
* Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15)
* /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1,
* 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/
C
IF (IFLAG .NE. 0) GO TO 5
C
Insert print statements here when NPRINT is positive.
C
RETURN
5 CONTINUE
DO 10 I = 1, M
TMP1 = I
TMP2 = 16 - I
TMP3 = TMP1
IF (I .GT. 8) TMP3 = TMP2
FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
10 CONTINUE
RETURN
END

Results obtained with different compilers or machines
may be slightly different.

FINAL L2 NORM OF THE RESIDUALS 0.9063596E-01
EXIT PARAMETER 1

FINAL APPROXIMATE SOLUTION

0.8241058E-01 0.1133037E+01 0.2343695E+01

For IOPT=2, FCN would be modified as follows to also calculate the full Jacobian when IFLAG=2.

SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)

C This is the form of the FCN routine if IOPT=2, that is, if the user calculates the full Jacobian.

INTEGER I,LDFJAC,M,N,IFLAG
DOUBLE PRECISION X(N),FVEC(M),FJAC(LDFJAC,N),Y(15)
DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8), * Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15) * /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1, * 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/

C IF (IFLAG .NE. 0) GO TO 5

C Insert print statements here when NPRINT is positive.

RETURN

5 CONTINUE

IF(IFLAG.NE.1) GO TO 20
DO 10 I = 1, M
TMP1 = I
TMP2 = 16 - I
TMP3 = TMP1
IF (I .GT. 8) TMP3 = TMP2
FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
10 CONTINUE
RETURN

C Below, calculate the full Jacobian.

C

20 CONTINUE

C DO 30 I = 1, M
TMP1 = I
TMP2 = 16 - I
TMP3 = TMP1
IF (I .GT. 8) TMP3 = TMP2
TMP4 = (X(2)*TMP2 + X(3)*TMP3)**2
FJAC(I,1) = -1.E0
FJAC(I,2) = TMP1*TMP2/TMP4
FJAC(I,3) = TMP1*TMP3/TMP4
30 CONTINUE
RETURN
For IOPT = 3, FJAC would be dimensioned as FJAC(3,3),
LDFJAC would be set to 3, and FCN would be written as
follows to calculate a row of the Jacobian when IFLAG=3.

SUBROUTINE FCN(IFLAG,M,N,X,FVEC,FJAC,LDFJAC)
C This is the form of the FCN routine if IOPT=3,
C that is, if the user calculates the Jacobian row by row.
INTEGER I,M,N,IFLAG
DOUBLE PRECISION X(N),FVEC(M),FJAC(N),Y(15)
DOUBLE PRECISION TMP1,TMP2,TMP3,TMP4
DATA Y(1),Y(2),Y(3),Y(4),Y(5),Y(6),Y(7),Y(8),
* Y(9),Y(10),Y(11),Y(12),Y(13),Y(14),Y(15)
* /1.4E-1,1.8E-1,2.2E-1,2.5E-1,2.9E-1,3.2E-1,3.5E-1,3.9E-1,
* 3.7E-1,5.8E-1,7.3E-1,9.6E-1,1.34E0,2.1E0,4.39E0/
C
IF (IFLAG .NE. 0) GO TO 5
C
C Insert print statements here when NPRINT is positive.
C
RETURN
5 CONTINUE
IF( IFLAG .NE. 1) GO TO 20
DO 10 I = 1, M
TMP1 = I
TMP2 = 16 - I
TMP3 = TMP1
IF (I .GT. 8) TMP3 = TMP2
FVEC(I) = Y(I) - (X(1) + TMP1/(X(2)*TMP2 + X(3)*TMP3))
10 CONTINUE
RETURN
C
Below, calculate the LDFJAC-th row of the Jacobian.
C
20 CONTINUE
I = LDFJAC
TMP1 = I
TMP2 = 16 - I
TMP3 = TMP1
IF (I .GT. 8) TMP3 = TMP2
TMP4 = (X(2)*TMP2 + X(3)*TMP3)**2
FJAC(1) = -1.E0
FJAC(2) = TMP1*TMP2/TMP4
FJAC(3) = TMP1*TMP3/TMP4
RETURN
END

***REFERENCES Jorge J. More, The Levenberg-Marquardt algorithm:
implementation and theory. In Numerical Analysis
Proceedings (Dundee, June 28 - July 1, 1977, G. A.
Watson, Editor), Lecture Notes in Mathematics 630,

***ROUTINES CALLED DNLS1, XERMSG

***REVISION HISTORY (YYMMDD)

800301 DATE WRITTEN
890831 Modified array declarations. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to XERMSG. (THJ)
920501 Reformatted the REFERENCES section. (WRB)
960918 Renamed PDA_DNLS1E and added STATUS argument (PWA)

***END PROLOGUE DNLS1E
PDA_DP1VLU
Use coefficients from PDA_DPOLFT to evaluate polynomial fit and its derivatives.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DP1VLU (L, NDER, X, YFIT, YP, A, STATUS)

***BEGIN PROLOGUE  PDA_DP1VLU
***PURPOSE  Use the coefficients generated by PDA_DPOLFT to evaluate the polynomial fit of degree L, along with the first NDER of its derivatives, at a specified point.
***LIBRARY  SLATEC
***CATEGORY  K6
***TYPE  DOUBLE PRECISION (PVALUE-S, PDA_DP1VLU-D)
***KEYWORDS  CURVE FITTING, LEAST SQUARES, POLYNOMIAL APPROXIMATION
***AUTHOR  Shampine, L. F., (SNLA)
       Davenport, S. M., (SNLA)
***DESCRIPTION

Abstract

The subroutine PDA_DP1VLU uses the coefficients generated by PDA_DPOLFT to evaluate the polynomial fit of degree L, along with the first NDER of its derivatives, at a specified point. Computationally stable recurrence relations are used to perform this task.

The parameters for PDA_DP1VLU are

Input -- ALL TYPE REAL variables are DOUBLE PRECISION
L - the degree of polynomial to be evaluated. L may be any non-negative integer which is less than or equal to NDEG, the highest degree polynomial provided by PDA_DPOLFT.
NDER - the number of derivatives to be evaluated. NDER may be 0 or any positive value. If NDER is less than 0, it will be treated as 0.
X - the argument at which the polynomial and its derivatives are to be evaluated.
A - work and output array containing values from last call to PDA_DPOLFT.

Output -- ALL TYPE REAL variables are DOUBLE PRECISION
YFIT - value of the fitting polynomial of degree L at X
YP - array containing the first through NDER derivatives of the polynomial of degree L. YP must be dimensioned at least NDER in the calling program.
STATUS - Returned error status.
       The status must be zero on entry. This routine does not check the status on entry.

***REFERENCES  L. F. Shampine, S. M. Davenport and R. E. Huddleston,
***ROUTINES CALLED  PDA_XERMSG
***REVISION HISTORY (YYMMDD)
740601  DATE WRITTEN
890531  Changed all specific intrinsics to generic. (WRB)
890911  Removed unnecessary intrinsics. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
900510 Convert XERRWV calls to PDA_XERMSG calls. (RWC)
920501 Reformatted the REFERENCES section. (WRB)
950404 Implement status. (HME)

***END PROLOGUE  PDA_DP1VLU
PDA_DPCOEF
Convert the PDA_DPOLFT coefficients to Taylor series form.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DPCOEF (L, C, TC, A, STATUS)

***BEGIN PROLOGUE  PDA_DPCOEF
***PURPOSE  Convert the PDA_DPOLFT coefficients to Taylor series form.
***LIBRARY  SLATEC
***CATEGORY  K1A1A2
***TYPE  DOUBLE PRECISION (PCOEFS, PDA_DPCOEFD)
***KEYWORDS  CURVE FITTING, DATA FITTING, LEAST SQUARES, POLYNOMIAL FIT
***AUTHOR  Shampine, L. F., (SNLA)
Davenport, S. M., (SNLA)
***DESCRIPTION

Abstract

PDA_DPOLFT computes the least squares polynomial fit of degree \( L \) as a sum of orthogonal polynomials. PDA_DPCOEF changes this fit to its Taylor expansion about any point \( C \), i.e. writes the polynomial as a sum of powers of \((X-C)\). Taking \( C=0 \) gives the polynomial in powers of \( X \), but a suitable non-zero \( C \) often leads to polynomials which are better scaled and more accurately evaluated.

The parameters for PDA_DPCOEF are

INPUT -- All TYPE REAL variables are DOUBLE PRECISION

\( L \) - Indicates the degree of polynomial to be changed to its Taylor expansion. To obtain the Taylor coefficients in reverse order, input \( L \) as the negative of the degree desired. The absolute value of \( L \) must be less than or equal to NDEG, the highest degree polynomial fitted by PDA_DPOLFT.

\( C \) - The point about which the Taylor expansion is to be made.

\( A \) - Work and output array containing values from last call to PDA_DPOLFT.

OUTPUT -- All TYPE REAL variables are DOUBLE PRECISION

\( TC \) - Vector containing the first \( LL+1 \) Taylor coefficients where \( LL=\text{ABS}(L) \). If \( L\text{.GT.0} \), the coefficients are in the usual Taylor series order, i.e.

\[
P(X) = TC(1) + TC(2) \times (X-C) + \ldots + TC(N+1) \times (X-C)^{N}
\]

If \( L\text{.LT.0} \), the coefficients are in reverse order, i.e.

\[
P(X) = TC(1) \times (X-C)^{N} + \ldots + TC(N) \times (X-C) + TC(N+1)
\]

\( STATUS \) - Returned error status.

The status must be zero on entry. This routine does not check the status on entry.

***REFERENCES  L. F. Shampine, S. M. Davenport and R. E. Huddleston,

***ROUTINES CALLED  PDA_DP1VLU
***REVISION HISTORY  (YMMDD)
740601  DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
920501 Reformatted the REFERENCES section. (WRB)
950404 Implement status. (HME)
950517 Return immediately if PDA_DP1VLU returns a status. (HME)

***END PROLOGUE PDA_DPCOEF
**PDA_DPLINT**

Produce the polynomial which interpolates a set of discrete data points.

**Origin:**
SLATEC / CAMSUN

**Implementation Status:**
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DPLINT (N, X, Y, C, STATUS)

***BEGIN PROLOGUE  PDA_DPLINT
***PURPOSE  Produce the polynomial which interpolates a set of discrete
data points.
***LIBRARY  SLATEC
***CATEGORY  E1B
***TYPE  DOUBLE PRECISION (POLINT-S, PDA_DPLINT-D)
***KEYWORDS  POLYNOMIAL INTERPOLATION
***AUTHOR  Huddleston, R. E., (SNLL)
***DESCRIPTION

Abstract
Subroutine PDA_DPLINT is designed to produce the polynomial which
interpolates the data (X(I),Y(I)), I=1,...,N. PDA_DPLINT sets up
information in the array C which can be used by subroutine PDA_DPOLVL
to evaluate the polynomial and its derivatives and by subroutine
PDA_DPOLCF to produce the coefficients.

Formal Parameters
*** All TYPE REAL variables are DOUBLE PRECISION ***
N - the number of data points  (N .GE. 1)
X - the array of abscissas (all of which must be distinct)
Y - the array of ordinates
C - an array of information used by subroutines
STATUS - Returned error status.
The status must be zero on entry. This
type does not check the status on entry.

******** Dimensioning Information *******
Arrays X,Y, and C must be dimensioned at least N in the calling
program.

***REFERENCES  L. F. Shampine, S. M. Davenport and R. E. Huddleston,
Curve fitting by polynomials in one variable, Report

***ROUTINES CALLED  PDA_XERMSG

***REVISION HISTORY  (YMMDD)
740601  DATE WRITTEN
891006  Cosmetic changes to prologue.  (WRB)
891006  REVISION DATE from Version 3.2
891214  Prologue converted to Version 4.0 format.  (BAB)
900315  CALLs to XERROR changed to CALLs to PDA_XERMSG.  (THJ)
920501  Reformatted the REFERENCES section.  (WRB)
950403  Implement status.  (HME)

***END PROLOGUE  PDA_DPLINT
PDA_DPOLCF
Coefficients of the polynomial fit (including Hermite polynomial fits) produced by PDA_DPLINT.

Origin:
SLATEC / CAMSUN
SUBROUTINE PDA_DPOLCF (XX, N, X, C, D, WORK)

***BEGIN PROLOGUE  PDA_DPOLCF
***PURPOSE  Compute the coefficients of the polynomial fit (including Hermite polynomial fits) produced by a previous call to POLINT.
***LIBRARY  SLATEC
***CATEGORY  E1B
***TYPE  DOUBLE PRECISION (POLCOF-S, PDA_DPOLCF-D)
***KEYWORDS  COEFFICIENTS, POLYNOMIAL
***AUTHOR  Huddleston, R. E., (SNLL)
***DESCRIPTION

Abstract
Subroutine PDA_DPOLCF computes the coefficients of the polynomial fit (including Hermite polynomial fits) produced by a previous call to PDA_DPLINT. The coefficients of the polynomial, expanded about XX, are stored in the array D. The expansion is of the form
\[ P(Z) = D(1) + D(2)(Z-XX) + D(3)\((Z-XX)^2\) + ... + D(N)\((Z-XX)^{(N-1)}\).\]

Between the call to PDA_DPLINT and the call to PDA_DPOLCF the variable N and the arrays X and C must not be altered.

***** INPUT PARAMETERS
*** All TYPE REAL variables are DOUBLE PRECISION ***

XX - The point about which the Taylor expansion is to be made.

N - ****
* N, X, and C must remain unchanged between the
X - * call to PDA_DPLINT and the call to PDA_DPOLCF.
C - ****

***** OUTPUT PARAMETER
*** All TYPE REAL variables are DOUBLE PRECISION ***

D - The array of coefficients for the Taylor expansion as explained in the abstract

***** STORAGE PARAMETER

WORK - This is an array to provide internal working storage. It must be dimensioned by at least 2*N in the calling program.

**** Note - There are two methods for evaluating the fit produced by PDA_DPLINT. You may call PDA_DPOLVL to perform the task, or you may call PDA_DPOLCF to obtain the coefficients of the Taylor expansion and then write your own evaluation scheme. Due to the inherent errors in the computations of the Taylor expansion from the Newton coefficients produced by PDA_DPLINT, much more accuracy may be expected by calling PDA_DPOLVL as opposed to writing your own scheme.
***REFERENCES (NONE)
***ROUTINES CALLED (NONE)
***REVISION HISTORY (YYMMDD)
  890213  DATE WRITTEN
  891006  Cosmetic changes to prologue. (WRB)
  891024  Corrected KEYWORD section. (WRB)
  891024  REVISION DATE from Version 3.2
  891214  Prologue converted to Version 4.0 format. (BAB)
***END PROLOGUE  PDA_DPOLCF
PDA_DPOLLFT
Weighted least-squares polynomial fit.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine and its subsidiaries will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DPOLFT (N, X, Y, W, MAXDEG, NDEG, EPS, R, IERR, A, + STATUS)

***BEGIN PROLOGUE  PDA_DPOLFT
***PURPOSE  Fit discrete data in a least squares sense by polynomials
           in one variable.
***LIBRARY  SLATEC
***CATEGORY  K1A1A2
***TYPE  DOUBLE PRECISION (POLFIT-S, PDA_DPOLFT-D)
***KEYWORDS  CURVE FITTING, DATA FITTING, LEAST SQUARES, POLYNOMIAL FIT
***AUTHOR  Shampine, L. F., (SNLA)
              Davenport, S. M., (SNLA)
              Huddleston, R. E., (SNLL)
***DESCRIPTION

Abstract

Given a collection of points X(I) and a set of values Y(I) which
 correspond to some function or measurement at each of the X(I),
 subroutine  PDA_DPOLFT  computes the weighted least-squares polynomial
 fits of all degrees up to some degree either specified by the user
 or determined by the routine. The fits thus obtained are in
 orthogonal polynomial form. Subroutine  PDA_DP1VLU  may then be
 called to evaluate the fitted polynomials and any of their
 derivatives at any point. The subroutine  PDA_DPCOEF  may be used to
 express the polynomial fits as powers of (X-C) for any specified
 point C.

The parameters for  PDA_DPOLFT  are

Input -- All  TYPE REAL  variables are  DOUBLE PRECISION

N  -  the number of data points. The arrays X, Y and W
     must be dimensioned at least  N  (N .GE. 1).
X  -  array of values of the independent variable. These
     values may appear in any order and need not all be
     distinct.
Y  -  array of corresponding function values.
W  -  array of positive values to be used as weights. If
     W(I) is negative,  PDA_DPOLFT  will set all the weights
     to 1.0, which means unweighted least squares error
     will be minimized. To minimize relative error, the
     user should set the weights to:  W(I) = 1.0/Y(I)**2,
     I = 1,...,N.
MAXDEG  -  maximum degree to be allowed for polynomial fit.
     MAXDEG  may be any non-negative integer less than  N.
     Note --  MAXDEG  cannot be equal to  N-1  when a
     statistical test is to be used for degree selection,
     i.e., when input value of  EPS  is negative.
EPS  -  specifies the criterion to be used in determining
     the degree of fit to be computed.
     (1) If  EPS  is input negative,  PDA_DPOLFT  chooses the
         degree based on a statistical F test of
         significance. One of three possible
significance levels will be used: .01, .05 or .10. If EPS=-1.0, the routine will automatically select one of these levels based on the number of data points and the maximum degree to be considered. If EPS is input as -.01, -.05, or -.10, a significance level of .01, .05, or .10, respectively, will be used.

(2) If EPS is set to 0., PDA_DPOLFT computes the polynomials of degrees 0 through MAXDEG.

(3) If EPS is input positive, EPS is the RMS error tolerance which must be satisfied by the fitted polynomial. PDA_DPOLFT will increase the degree of fit until this criterion is met or until the maximum degree is reached.

Output -- All TYPE REAL variables are DOUBLE PRECISION

NDEG - degree of the highest degree fit computed.
EPS - RMS error of the polynomial of degree NDEG.
R - vector of dimension at least N containing values of the fit of degree NDEG at each of the X(I).
Except when the statistical test is used, these values are more accurate than results from subroutine PDA_DP1VLU normally are.
IERR - error flag with the following possible values.
1 -- indicates normal execution, i.e., either
(1) the input value of EPS was negative, and the computed polynomial fit of degree NDEG satisfies the specified F test, or
(2) the input value of EPS was 0., and the fits of all degrees up to MAXDEG are complete, or
(3) the input value of EPS was positive, and the polynomial of degree NDEG satisfies the RMS error requirement.
2 -- invalid input parameter. At least one of the input parameters has an illegal value and must be corrected before PDA_DPOLFT can proceed. Valid input results when the following restrictions are observed
N .GE. 1
0 .LE. MAXDEG .LE. N-1 for EPS .GE. 0.
0 .LE. MAXDEG .LE. N-2 for EPS .LT. 0.
W(I)=-1.0 or W(I) .GT. 0., I=1,...,N.
3 -- cannot satisfy the RMS error requirement with a polynomial of degree no greater than MAXDEG. Best fit found is of degree MAXDEG.
4 -- cannot satisfy the test for significance using current value of MAXDEG. Statistically, the best fit found is of order NORD. (In this case, NDEG will have one of the values: MAXDEG-2, MAXDEG-1, or MAXDEG). Using a higher value of MAXDEG may result in passing the test.
A - work and output array having at least 3N+3MAXDEG+3 locations
STATUS - Returned error status.
The status must be zero on entry. This
routine does not check the status on entry.

Note - PDA_DPOLFT calculates all fits of degrees up to and including NDEG. Any or all of these fits can be evaluated or expressed as powers of (X-C) using PDA_DP1VLU and PDA_DPCOEF after just one call to PDA_DPOLFT.


***ROUTINES CALLED PDA_DP1VLU, PDA_XERMSG

***REVISION HISTORY (YYMMDD)
740601 DATE WRITTEN
890531 Changed all specific intrinsics to generic. (WRB)
891006 Cosmetic changes to prologue. (WRB)
891006 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
900911 Added variable YP to DOUBLE PRECISION declaration. (WRB)
920501 Reformatted the REFERENCES section. (WRB)
920527 Corrected erroneous statements in DESCRIPTION. (WRB)
950404 Implement status. (HME)
950517 Return immediately if PDA_DP1VLU returns a status. (HME)

***END PROLOGUE PDA_DPOLFT
PDA_DPOLVL
Evaluate polynomial and its derivatives as produced by
PDA_DPLINT.

Origin:
SLATEC / CAMSUN
SUBROUTINE PDA_DPOLVL (NDER, XX, YFIT, YP, N, X, C, WORK, IERR)

***BEGIN PROLOGUE  PDA_DPOLVL
***PURPOSE Calculate the value of a polynomial and its first NDER
derivatives where the polynomial was produced by a previous
call to PDA_DPLINT.
***LIBRARY  SLATEC
***CATEGORY  E3
***TYPE  DOUBLE PRECISION (POLYVL-S, PDA_DPOLVL-D)
***KEYWORDS  POLYNOMIAL EVALUATION
***AUTHOR  Huddleston, R. E., (SNLL)
***DESCRIPTION

Abstract -
Subroutine PDA_DPOLVL calculates the value of the polynomial and
its first NDER derivatives where the polynomial was produced by
a previous call to PDA_DPLINT.

The variable N and the arrays X and C must not be altered
between the call to PDA_DPLINT and the call to PDA_DPOLVL.

******  Dimensioning Information ******

YP  must be dimensioned by at least NDER
X   must be dimensioned by at least N (see the abstract )
C   must be dimensioned by at least N (see the abstract )
WORK must be dimensioned by at least 2*N if NDER is .GT. 0.

*** Note ***
If NDER=0, neither YP nor WORK need to be dimensioned variables.
If NDER=1, YP does not need to be a dimensioned variable.

******  Input parameters
***  All TYPE REAL variables are DOUBLE PRECISION ***
NDER - the number of derivatives to be evaluated
XX  - the argument at which the polynomial and its derivatives
        are to be evaluated.
N   - *****
    *  N, X, and C must not be altered between the call
X   - *
    to PDA_DPLINT and the call to PDA_DPOLVL.
C   - *****

******  Output Parameters
***  All TYPE REAL variables are DOUBLE PRECISION ***
YFIT - the value of the polynomial at XX
YP  - the derivatives of the polynomial at XX. The derivative of
    order J at XX is stored in  YP(J) , J = 1,...,NDER.
IERR - Output error flag with the following possible values.
   = 1  indicates normal execution

***** Storage Parameters

WORK = this is an array to provide internal working storage for
       PDA_DPOLVL. It must be dimensioned by at least 2*N if NDER is
       .GT. 0. If NDER=0, WORK does not need to be a dimensioned
       variable.

***REFERENCES  L. F. Shampine, S. M. Davenport and R. E. Huddleston,
       Curve fitting by polynomials in one variable, Report

***ROUTINES CALLED  (NONE)

***REVISION HISTORY  (YYMMDD)
    740601  DATE WRITTEN
    890531  Changed all specific intrinsics to generic.  (WRB)
    891006  Cosmetic changes to prologue.  (WRB)
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    891214  Prologue converted to Version 4.0 format.  (BAB)
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***END PROLOGUE  PDA_DPOLVL
PDA_DQED
Solves bounded nonlinear least squares and nonlinear equations.

Origin:
NETLIB/OPT
SUBROUTINE PDA_DQED ( PDA_DQEDEV, MEQUA, NVARS, MCON, IND, BL, BU,
: X, FJAC, LDFJAC, FNORM, IGO, IOPT, ROPT,
: IWA, WA )

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***CATEGORY NO. K1b,K1b1a2,K1b2a
***KEYWORDS NONLINEAR LEAST SQUARES, SIMPLE BOUNDS,
LINEAR CONSTRAINTS

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***PURPOSE SOLVE NONLINEAR LEAST SQUARES AND NONLINEAR
EQUATIONS. USER PROVIDES SIMPLE BOUNDS, LINEAR
CONSTRAINTS AND EVALUATION CODE FOR THE FUNCTIONS.

***LONG DESCRIPTION

SUBROUTINE PDA_DQED (PDA_DQEDEV, MEQUA, NVARS, MCON, IND, BL, BU, X,
* FJ, LDFJ, RNORM, IGO, IOPT, ROPT,
* IWORK, WORK)

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1. Introduction
---------------

The Fortran subprogram, PDA_DQED(), solves the constrained nonlinear
least squares problem:

Minimize the sum of squares of MEQUA (generally nonlinear) equations,

\[ f(x) = 0, I=1,...,MEQUA \quad \text{Eq. (1)} \]

where \( x \) is a (vector) set of NVARS unknowns. (The vector
function with these MEQUA components is called \( f(x) \) in the
discussion that follows.) The components of \( x \) may have upper
and lower bounds given by the user. (In fact all of the
possible cases, no bounds, bounds at one end only, or upper and
lower bounds can be specified.) Linear constraints on the
unknowns, more general than simple bounds, can also be given.
These constraints can be of the equality or inequality type:

\[ a_1 x + \ldots + a_{L1} x_{L1} + \ldots + a_{NL} x_{NL} = y, \ L = 1, \ldots, MCON, \]

\[ L, NVARS \]

Eq. (2)

with bounds specified on the \( y \), again given by the user. The \( L \) constraints can actually be slightly nonlinear. In this case the constraints can be described as:

\[ g(x) = y, \ L = 1, \ldots, MCON, \]

\[ L, L \]

Eq. (2')

where bounds are specified on each \( y \). The functions \( g(x) \) must be defined for all \( x \) in the set described by the simple bounds. Experienced users may wish to turn directly to Examples 1 and 2, listed below, before reading the subprogram documentation. There is no size relation required for the problem dimensions MEQUA, NVARS, and MCON except that MEQUA and NVARS are both positive, and MCON is nonnegative.

This code package will do a decent job of solving most nonlinear least squares problems that can be expressed as Eqs. (1) and (2) above, provided that continuous derivatives of the functions with respect to the parameters can be computed. This can also include problems where the derivatives must be computed using some form of numerical differentiation. Numerical differentiation is not provided with this software for solving nonlinear least squares problems. Refer to the subprogram JACG for numerical differentiation. (Note: D. Salane has this submitted to TOMS. It is not included here.)

The authors also plan to develop methods that will do a much better job of coping with constraints more general than the essentially linear ones indicated above in Eqs. (2)-(2'). There are nonlinear least squares problems with innocent looking but highly nonlinear constraints where this package will fail to work. The authors also hope to reduce the overhead required by the software. This high overhead is due primarily to the method used to solve the inner-loop quadratic model problem. The authors recommend that users consider using the option number 14, described below, to suppress use of the quadratic model. The user may find that the software works quite well without the quadratic model. This may be important when the function and derivatives evaluations are not expensive but many individual problems are being solved.

There are two fundamental ways to use the subprogram PDA_DQED(). The most straightforward way is to make one Fortran CALL to the subprogram and obtain values for the unknowns, \( x \). The user provides a subprogram PDA_DQEDEV(), described below, that gives the subprogram PDA_DQED() values of the functions \( f(x) \) and \( g(x) \), and the derivative or Jacobian matrices for \( f(x) \) and \( g(x) \) at each
desired point \( x \). This usage is called 'forward communication.' An alternate way to use the subprogram is to provide an option that allows the user to communicate these values by 'reverse communication.' The subprogram returns to the calling program unit and requests values for \( f(x) \) and \( g(x) \), and the Jacobian matrices for \( f(x) \) and \( g(x) \) for a given value of \( x \). (This framework is often required in applications that have complicated algorithmic requirements for evaluation of the functions.) An example using both 'forward' and 'reverse' communication is provided below (see Remarks on the Usage Examples) for least squares fitting of two exponential functions to five data points.

2. Calling Sequence Explained

There are arrays used by the subprogram that must have dimensions equivalent to the following declarations.

\[
\begin{align*}
&\text{INTEGER MEQUA, NVARS, MCON, LDFJ, IGO} \\
&\text{INTEGER IND(NVARS+MCON), IOPT(LIOPT), IWORK(LIWORK)} \\
&\text{DOUBLE PRECISION BL(NVARS+MCON), BU(NVARS+MCON), X(NVARS), RNORM,} \\
&\quad \text{*ROPT(LROPT), FJ(LDFJ,NVARS+1), WORK(LWORK)} \\
&\text{EXTERNAL PDA_DQEDEV}
\end{align*}
\]

The array dimensions must satisfy the bounds:

\[
\begin{align*}
&\text{LIOPT .ge. Number required for options in use.} \\
&\text{LROPT .ge. Number required for options in use.} \\
&\text{LDFJ .ge. MEQUA+MCON,}
\end{align*}
\]

The array dimensions for the arrays \( \text{IWORK(*)} \) and \( \text{WORK(*)} \) can change if either option 14 or option 15 are in use. For use in the formulas, define:

\[
\begin{align*}
&\text{MC=MCON} \\
&\text{ME=MEQUA} \\
&\text{NV=NVARS} \\
&\text{MX=MAX(MEQUA,NVARS)} \\
&\text{If the user is not using option 15, then NT=5.} \\
&\text{If the user is using option 15, then NT=new number, must be .ge. 2.} \\
&\text{If the user is not using option 14, then NA=MC+2*NV+NT.} \\
&\text{If the user is using option 14, then NA=MC+NV+1.}
\end{align*}
\]

In terms of these values defined above,

\[
\begin{align*}
&\text{LIWORK .ge. } 3*MC+9*NV+4*NT+NA+10 \\
&\text{LWORK .ge. } NA*(NA+4)+NV*(NT+33)+(ME+MX+14)*NT+9*MC+26
\end{align*}
\]

The subprogram \( \text{PDA_DQEDEV} \) must be declared in a Fortran \text{EXTERNAL} statement:
EXTERNAL PDA_DQEDEV

Initialize the values of the parameters:
MEQUA, NVARS, MCON, IND(*), BL(*), BU(*), X(*), LDFJ,
IOPT(*), IWORK(1), IWORK(2),

CALL PDA_DQED (DQEDEV, MEQUA, NVARS, MCON, IND, BL, BU, X,
* FJ, LDFJ, RNORM, IGD, IOPT, ROPT,
* IWORK, WORK)

Subprogram parameters:

PDA_DQEDEV (Input)
----------
This is the name of a subprogram that the user will usually supply for evaluation of the values of the constraints and model, and the derivatives of these functions. The user must provide this subprogram unless 'reverse communication' is used. A model for writing the subprogram PDA_DQEDEV() is provided under the heading Example 1 Using Forward Communication, listed below. Users may find it convenient to modify this model subprogram when writing a subprogram for their own application. If 'reverse communication' is used, the user does not need to write a stub or dummy subroutine named PDA_DQEDEV(). All that is required is to declare exactly this name in an EXTERNAL statement. The code package has a dummy subroutine PDA_DQEDEV() that will be used in the linking or load step. Example 2 Using Reverse Communication, listed below, illustrates this detail.

MEQUA, NVARS, MCON (Input)
-----------
Respectively they are: The number of least squares equations, the number of unknowns or variables, and the number of general constraints for the solution, not including simple bounds. The values of MEQUA and NVARS must be positive; the value of MCON must be nonnegative. Other values for these parameters are errors.

IND(*),BL(*),BU(*) (Input)
-----------
These arrays describe the form of the simple bounds that the components of x are to satisfy. Components numbered 1,...,NVARS are used to describe the form of the simple bounds that the unknown are to satisfy. Components numbered NVARS+1,...,NVARS+MCON are used to describe the form of the general MCON linear constraints. The first NVARS components of IND(*) indicate the type of simple bounds that the solution is to satisfy. The corresponding entries of BL(*) and BU(*) are the bounding value. The only values of IND(*) allowed are 1,2,3 or 4. Other values are errors. Specifically:

IND(J)=1, if x .ge. BL(J) is required; BU(J) is not used.
J
=2, if x .le. BU(J) is required; BL(J) is not used.
J
=3, if \( x \geq BL(J) \) and \( x \leq BU(J) \) is required.

J
=4, if no bounds on \( x \) are required;

J
BL(*), BU(*) are not used.

General linear constraints of the form shown in Eq. (2) require that bounds be given for the linear functions \( y \). Specifically:

L
IND(NVARS+L)=1, if \( y \geq BL(NVARS+L) \) is required; BU(*) is not needed.

L
=2, if \( y \leq BU(NVARS+L) \) is required; BL(*) is not needed.

L
=3, if \( y \geq BL(NVARS+L) \) and \( y \leq BU(NVARS+L) \)

L
=4, if no bounds on \( y \) are required;

L
BL(*), BU(*) are not used.

The values of the bounds for the unknowns may be changed by the user during the evaluation of the functions \( f(x) \) and \( g(x) \) and their Jacobian matrices.

X(*), FJ(*, *), LDFJ (Input and Output, except LDFJ which is Input)
-----------------------------

The array \( X(*) \) contains the NVARS values, \( x \), where the functions \( f(x) \) and \( g(x) \) and their Jacobian matrices will be evaluated by the subprogram \( PDA_DQED() \). After the computation has successfully completed, the array \( X(*) \) will contain a solution, namely the unknowns of the problem, \( x \). (Success is determined by an appropriate value for IGO. This parameter is described below.) Initially the array \( X(*) \) must contain a starting guess for the unknowns of the problem, \( x \). The initial values do not need to satisfy the constraints or the bounds. If they do not satisfy the bounds, then the point will be simply projected onto the bounds as a first step. The first linear change to the values of \( x \) must satisfy the general constraints. (It is here that the assumption of their linearity is utilized.)

The Fortran two-dimensional array \( FJ(*, *) \) is used to store the linear constraints of Eq. (2) (or more generally the Jacobian matrix of the functions \( g(x) \) with respect to the variables \( x \)), and the Jacobian matrix of the function \( f(x) \). The Jacobian matrix of the (linear) constraints is placed in rows 1,...,MCON. The Jacobian matrix of \( f(x) \) is placed in rows MCON+1,..., MCON+MEQUA. The parameter LDFJ is the leading or row dimension of the array \( FJ(*, *) \). Normally the array \( FJ(*, *) \) is assigned values by the user when the nonlinear solver requests evaluations of the constraints \( g(x) \) and the function \( f(x) \).
together with the Jacobian matrices \( G(x) \) and \( J(x) \). The values of the constraint functions \( g(x) \) are placed in the array \( L \) \( FJ(L,NVARS+1), \) \( L=1,...,MCON. \) The values of the model functions \( f(x) \) are placed in the array at entries \( FJ(MCON+I,NVARS+1), \) \( I=1,...,MEQUA. \) Note that the second dimension of \( FJ(*,*) \) must be at least \( NVARS+1 \) in value.

RNORM (Output)
-----
This is the value of the Euclidean length or square root of sums of squares of components of the function \( f(x) \) after the approximate solution, \( x \), has been found. During the computation it is updated and equals the best value of the length of \( f(x) \) that has been found.

IGO (Output; it can be an Input if interrupting the code)
---
This flag directs user action and informs the user about the type of results obtained by the subprogram. The user may find it convenient to treat the cases \( \text{abs}(IGO) \leq 1 \) the same as \( IGO=1. \) This has no effect on the solution process.

The user can interrupt the computation at any time, obtaining the best values of the vector \( x \) up to this point, by setting \( IGO \) to any value \( > 1 \) and then return control to PDA_DQED(). For example if a calculation must be done in a certain length of time, the user can, as the end of the time draws near, set \( IGO=20 \) and return control to PDA_DQED(). It is important that this method be used to stop further computing, rather than simply proceeding. The reason for this is that certain flags in PDA_DQED() must be reset before any further solving on subsequent problems can take place. The value of \( IGO > 1 \) used to interrupt the computation is arbitrary and is the value of \( IGO \) returned. If values of \( IGO = 2,...,18 \) are used to flag this interrupt, they do not mean the same thing as indicated below. For this reason the value \( IGO=20 \) is recommended for signaling interrupts in PDA_DQED(). Another situation that may occur is the request for an evaluation of the functions and derivatives at a point \( x \) where these can't be evaluated. If this occurs, set \( IGO=99 \) and return control to PDA_DQED(). This will have the effect of defining the derivatives to be all zero and the functions to be 'large.' Thus a reduction in the trust region around the current best estimate will occur. Assigning the value \( IGO=99 \) will not cause PDA_DQED() to stop computing.

=0 Place the value of \( f(x) \) in \( FJ(MCON+*,NVARS+1). \) If 'reverse communication' is being used, CALL PDA_DQED() again. If 'forward communication' is being used, do a RETURN.

=1 or (-1) Evaluate the Jacobians for the functions \( g(x) \) and \( f(x) \) as well as evaluating \( g(x) \) and \( f(x) \). Use the vector \( x \) that is now in the array \( X(*) \) as the values where this
evaluation will be performed. Place the Jacobian matrix for \( g(x) \) in the first \( MCON \) rows of \( FJ(*,*) \). Place the Jacobian matrix for \( f(x) \) in rows \( MCON+1,...,MCON+MEQUA \) in \( FJ(*,*) \). Place the value of \( g(x) \) in \( FJ(*,NVARS+1) \). Place the value of \( f(x) \) in \( FJ(MCON+*,NVARS+1) \).

(Users who have complicated functions whose derivatives cannot be computed analytically may want to use the numerical differentiation subroutine JAGC. This is available on the SLATEC library.)

If 'reverse communication' is being used, CALL PDA_DQED() again. If 'forward communication' is being used, do a RETURN.

A value \( IGO=(-1) \) flags that that the number of terms in the quadratic model is being restricted by the amount of storage given for that purpose. It is suggested, but it is not required, that additional storage be given for the quadratic model parameters. See the following description of The Option Array, option number 15, for the way to designate more storage for this purpose.

\[ =2 \] The function \( f(x) \) has a length less than \( TOLF \). This is the value for \( IGO \) to be expected when an actual zero value of \( f(x) \) is anticipated. See the description of The Option Array for the value.

\[ =3 \] The function \( f(x) \) has reached a value that may be a local minimum. However, the bounds on the trust region defining the size of the step are being hit at each step. Thus the situation is suspect. (Situations of this type can occur when the solution is at infinity in some of the components of the unknowns, \( x \). See the description of The Option Array for ways to avoid this value of output value of \( IGO \).

\[ =4 \] The function \( f(x) \) has reached a local minimum. This is the value of \( IGO \) that is expected when a nonzero value of \( f(x) \) is anticipated. See the description of The Option Array for the conditions that have been satisfied.

\[ =5 \] The model problem solver has noted a value for the linear or quadratic model problem residual vector length that is \( \geq \) the current value of the function, i.e. the Euclidean length of \( f(x) \). This situation probably means that the evaluation of \( f(x) \) has more uncertainty or noise than is possible to account for in the tolerances used to note a local minimum. The value for \( x \) is suspect, but a minimum has probably been found.

\[ =6 \] A small change (absolute) was noted for the vector \( x \). A full model problem step was taken. The condition for \( IGO=4 \) may also be satisfied, so that a minimum has been found. However, this test is made before the test for \( IGO=4 \).

\[ =7 \] A small change (relative to the length of \( x \)) was noted
for the vector $x$. A full model problem step was taken. The
condition for $IGO=4$ may also be satisfied, so that a minimum has
been found. However, this test is made before the test for
$IGO=4$.

8 More than ITMAX iterations were taken to obtain the
solution. The value obtained for $x$ is suspect, although it is
the best set of $x$ values that occurred in the entire
computation. See the description of The Option Array for
directions on how to increase this value. (Note that the
nominal value for ITMAX, 75, is sufficient to solve all of the
nonlinear test problems described in Ref. (2).)

9-18 Errors in the usage of the subprogram were noted.
The exact condition will be noted using an error processor that
prints an informative message unless this printing has been
suppressed. A minimum value has not been found for $x$. The
relation between $IGO$ and the error number are $IGO=NERR + 8$.
Here $NERR$ is the identifying number. See below, Error Messages
for PDA_DQED().

The Option Array
--- ------ -----

Glossary of Items Modified by Options. Those items with Nominal
Values listed can be changed.

<table>
<thead>
<tr>
<th>Names</th>
<th>Nominal Values</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>FC</td>
<td>Current value of length of $f(x)$.</td>
<td></td>
</tr>
<tr>
<td>FB</td>
<td>Best value of length of $f(x)$.</td>
<td></td>
</tr>
</tbody>
</table>
| FL    | Value of length of $f(x)$ at the
     | previous step. |
| PV    | Predicted value of length of $f(x)$,
     | after the step is taken, using the
     | approximating model. |

The quantity 'eps', used below, is the machine precision
parameter. Its value is obtained by a call to the Bell Labs.
Port subprogram D1MACH(4). It is machine dependent.

$\text{TOLF} = \text{sqrt}(\text{eps})$ Tolerance for stopping when
$FC .le. \text{TOLF}$. 

$\text{TOLD} = \text{sqrt}(\text{eps})$ Tolerance for stopping when
change to $x$ values has length
$.le. \text{TOLD}$. 

$\text{TOLX} = \text{sqrt}(\text{eps})$ Tolerance for stopping when
change to $x$ values has length
$.le. \text{TOLX} \times \text{length of } x$ values. 

$\text{TOLSNR} = 1.D-5$ Tolerance used in stopping
condition $IGO=4$. Explained below. 

$\text{TOLP} = 1.D-5$ Tolerance used in stopping
condition $IGO=4$. Explained below.
(The conditions (abs(FB-PV).le.TOLSNR*FB and abs(FC-PV).le.TOLP*FB) and (ABS(FC-FL).le.TOLSNR*FB) together with taking a full model step, must be satisfied before the condition IGO=4 is returned. Decreasing any of the values for TOLF, TOLD, TOLX, TOLSNR, or TOLP will likely increase the number of iterations required for convergence.)

COND  30. Largest condition number to allow when solving for the quadratic model coefficients. Increasing this value may result in more terms being used in the quadratic model.

TOLUSE sqrt(eps) A tolerance that is used to avoid values of x in the quadratic model’s interpolation of previous points. Decreasing this value may result in more terms being used in the quadratic model.

ITMAX  75 The number of iterations to take with the algorithm before giving up and noting it with the value IGO=8.

IPRINT  0 Control the level of printed output in the solver. A value of IPRINT.gt.0 will result in output of information about each iteration. The output unit used is obtained using the Bell Labs. Port subprogram, i.e. I1MACH(2).

LEVEL  1 Error processor error level. See the SLATEC library documentation for XERROR() for an explanation.

NTERMS  5 One more than the maximum number of terms used in the quadratic model.

IOPT(*) (Input)

In order to use the option array technique to change selected data within a subprogram, it is necessary to understand how this array is processed within the software. Let LP designate the processing pointer that moves to positions of the IOPT(*) array. Initially LP=1, and as each option is noted and changed, the value of LP is updated. The values of IOPT(LP) determine what options get changed. The amount that LP changes is known by the software to be equal to the value two except for two options. These exceptional cases are the last option (=99) and the 'leap' option (=13) which advances LP by the value in IOPT(LP+1). A negative value for IOPT(LP) means that this option is not to be changed. This aids the programmer in using options; often the code for using an option can be in the calling program but a negative value of the option number avoids rewriting code.
Option Usage Example
-------- ------ -------
In the Fortran code fragment that follows, an example is given where we change the value of TOLF and decrease the maximum number of iterations allowed from 75 to 30. In this example the dimensions of IOPT(*) and ROPT(*) must satisfy:

```
DOUBLE PRECISION ROPT(01)
INTEGER IOPT(005)
.
.
.
C SET THE OPTION TO CHANGE THE VALUE OF TOLF.
IOPT(01)=4
C THE NEXT ENTRY POINTS TO THE PLACE IN ROPT(*) WHERE C THE NEW VALUE OF TOLF IS LOCATED.
IOPT(02)=1
C THIS IS THE NEW VALUE OF TOLF. THE SPECIFIC VALUE C 1.D-9 IS USED HERE ONLY FOR ILLUSTRATION.
ROPT(01)=1.D-9
C CHANGE THE NUMBER OF ITERATIONS.
IOPT(03)=2
C THIS NEXT ENTRY IS THE NEW VALUE FOR THE MAXIMUM NUMBER OF C ITERATIONS.
IOPT(04)=30
C THIS NEXT OPTION IS A SIGNAL THAT THERE ARE NO MORE C OPTIONS.
IOPT(05)=99
.
.
.
CALL PDA_DQED()
.
.
.
Option Values   Explanation
-------- ------ --------------------
=99           There are no more options to change. Normally this is the first and only option that a user needs to specify, and it can be simply IOPT(01)=99. The total dimension of IOPT(*) must be at least 17, however. This can lead to a
hard-to-find program bug if the dimension is too small.

= 1 Change the amount of printed output. The next value of IOPT(*) is the print level desired, IPRINT. Any value of IPRINT .gt. 0 gives all the available output.

= 2 Change the value of ITMAX. The next value of IOPT(*) is the value of ITMAX desired.

= 3 Pass prior determined bounds for the box containing the initial point. This box is the trust region for the first move from the initial point. The next entry in IOPT(*) points to the place in ROPT(*) where the NVARS values for the edges of the box are found.

= 4 Change the value of TOLF. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLF is found.

= 5 Change the value of TOLX. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLX is found.

= 6 Change the value of TOLD. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLD is found.

= 7 Change the value of TOLSRN. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLSRN is found.

= 8 Change the value of TOLP. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLP is found.

= 9 Change the value of TOLUSE. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of TOLUSE is found.

= 10 Change the value of COND. The next entry of IOPT(*) points to the place in ROPT(*) where the new value of COND is found.

= 11 Change the value of LEVEL. The next entry of IOPT(*) is the new value of LEVEL.

= 12 Pass an option array to the subprogram PDA_DQEDGN() used as the inner loop solver for the model problem. The next entry of IOPT(*) is the starting location for the option array for
PDA_DQEDGN() within the array IOPT(*). Thus the option array for PDA_DQEDGN() must be a part of the array IOPT(*).

=13  Move (or leap) the processing pointer LP for the option array by the next value in IOPT(*)).

=14  Change a logical flag that suppresses the use of the quadratic model in the inner loop. Use the next value in IOPT(*) for this flag. If this value = 1, then never use the quadratic model. (Just use the linear model). Otherwise, use the quadratic model when appropriate. This option decreases the amount of scratch storage as well as the computing overhead required by the code package. A user may want to determine if the application really requires the use of the quadratic model. If it does not, then use this option to save both storage and computing time.

=15  Change, NTERMS, the maximum number of array columns that can be used for saving quadratic model data. (The value of NTERMS is one more than the maximum number of terms used.) Each unit increase for NTERMS increases the required dimension of the array WORK(*) by 2*MEQUA+NVARS. Use the value in IOPT(LP+1) for the new value of NTERMS. Decreasing this value to 2 (its minimum) decreases the amount of storage required by the code package.

=16  Change a logical flag so that 'reverse communication' is used instead of 'forward communication.' Example EX01, listed below, uses 'forward communication.' Example EX02, also listed below, uses 'reverse communication.' Use the next value in IOPT(*) for this flag. If this value = 1, then use 'reverse communication.' Otherwise, use 'forward communication.' WARNING: This usage may not work unless the operating system saves variables between subroutine calls to PDA_DQED.

=17  Do not allow the flag IGO to return with the value IGO=3. This means that convergence will not be claimed unless a full model step is taken. Normal output values will then be IGO = 2, 4, 6 or 7. Use the next value in IOPT(*) for this flag. If this value = 1, then force a full model step. Otherwise, do not force a full model step if small steps are noted.

IWORK(*), WORK(*) (Input and Output)
These are scratch arrays that the software uses for storage of intermediate results. It is important not to modify the contents of this storage during the computation.

The array locations IWORK(1) and IWORK(2) must contain the actual lengths of the arrays WORK(*) and IWORK(*) before the call to the subprogram. These array entries are replaced by the actual amount of storage required for each array. If the amount of storage for either array is too small, an informative error message will be printed, and the value IGO=13 or 14 returned.

The user may find it useful to let the subprogram PDA_DQED() return the amounts of storage required for these arrays. For example set IWORK(1)=1, IWORK(2)=1. The subprogram will return with IGO=13, IWORK(1)=required length of WORK(*), and IWORK(2)=required length of IWORK(*). (Appropriate error messages will normally be printed.)

3. Remarks on the Usage Examples

The following complete program units, EX01 and EX02, show how one can use the nonlinear solver for fitting exponential functions to given data. These examples are calculations that match two terms of an exponential series to five given data points. There are some subtle points about exponential fitting that are important to note. First, the signs of the exponential arguments are restricted to be nonpositive. The size of the arguments should not be much larger than the start of the time data (reciprocated). This is the reason the lower bounds are set a bit less than the reciprocal of the time value. In many applications that require exponential modeling this is a natural assumption. The nonlinear solver allows these bounds on the arguments explicitly. In addition, the coefficients are constrained to be nonnegative. These bounds are harder to justify. The idea is to avoid the situation where a coefficient is very large and negative, and the corresponding exponential argument is also large and negative. The resulting contribution to the series may be very small, but its presence is spurious. Finally, the single general linear constraint that keeps the arguments separated (by 0.05 in this example) is used for two purposes. First, it naturally orders these values so that the first one is algebraically largest. Second, this constraint moves the parameters from the local minimum corresponding to the initial values used in the examples. This constraint also retains the validity of the model function h(t) = w*exp(x*t) + y*exp(z*t). Namely, if the arguments are allowed to coalesce to the same value, then the model itself must change. The form of the model can become h(t) = (a+b*t)*exp(c*t) or h(t) = d*exp(e*t). Either one could occur, and the choice is problem dependent.

Example 1 Using Forward Communication

---

PROGRAM EX01
Illustrate the use of the Hanson-Krogh nonlinear least squares solver for fitting two exponentials to data.

The problem is to find the four variables $x(1),\ldots,x(4)$ that are in the model function

$$h(t) = x(1)\cdot\exp(x(2)\cdot t) + x(3)\cdot\exp(x(4)\cdot t)$$

There are values of $h(t)$ given at five values of $t$, $t=0.05, 0.1, 0.4, 0.5,$ and $1.0$.

We also have problem constraints that $x(2), x(4) \leq 0$, $x(1), x(3) \geq 0$, and a minimal separation of $0.05$ between $x(2)$ and $x(4)$. Nothing more about the values of the parameters is known except that $x(2), x(4)$ are approximately $\geq 1/\min(t)$.

Thus we have no further knowledge of their values.

For that reason all of the initial values are set to zero.

Dimension for the nonlinear solver.

DOUBLE PRECISION $FJ(6,5),BL(5),BU(5),X(4),ROPT(001),WA(640)$

EDIT on 950228-1300:
DOUBLE PRECISION $RNORM$
INTEGER $IND(5),IOPT(24),IWA(084)$

EXTERNAL PDA_DQEDEX

DATA $LDFJ,LWA,LIWA/6,640,084/$

MCON = 1
MEQUA = 5
NVARS = 4

Define the constraints for variables.

$BL(1) = 0.$
$BL(2) = -25.$
$BU(2) = 0.$
$BL(3) = 0.$
$BL(4) = -25.$
$BU(4) = 0.$

Define the constraining value (separation) for the arguments.

$BL(5) = 0.05$

Define all of the constraint indicators.

$IND(1) = 1$
$IND(2) = 3$
$IND(3) = 1$
$IND(4) = 3$
$IND(5) = 1$

Define the initial values of the variables.

We don’t know anything more, so all variables are set zero.

DO 10 J = 1,NVARS
  $X(J) = 0.00$
10 CONTINUE

Tell how much storage we gave the solver.

$IWA(1) = LWA$
$IWA(2) = LIWA$

No additional options are in use.

IOPT(01) = 99
CALL PDA_DQED(PDA_DQEDEX,MEQUA,NVARS,MCON,IND,BL,BU,X,FJ,LDFJ,RNORM,IGO, 
   IOUT,IOPT,ROPT,IWA,WA)
NOUT = 6
WRITE (NOUT,9001) (X(J),J=1,NVARS)
WRITE (NOUT,9011) RNORM
WRITE (NOUT,9021) IGO
STOP

9001 FORMAT (' MODEL IS H(T) = X(1)*EXP(-T*X(2)) + X(3)*EXP(T*X(4))',/, 
   ' X(1),X(2),X(3),X(4) = ',/,4F12.6)
9011 FORMAT (' RESIDUAL AFTER THE FIT = ',1PD12.4)
9021 FORMAT (' OUTPUT FLAG FROM SOLVER =',17X,I6)
END

SUBROUTINE PDA_DQEDEX(X,FJ,LDFJ,IGO,IOPT,ROPT)
C This is the subprogram for evaluating the functions
C and derivatives for the nonlinear solver, PDA_DQED.
C
C The user problem has MCON constraint functions,
C MEQUA least squares equations, and involves NVARS
C unknown variables.
C
C When this subprogram is entered, the general (near)
C linear constraint partial derivatives, the derivatives
C for the least squares equations, and the associated
C function values are placed into the array FJ(*,*).
C All partials and functions are evaluated at the point
C in X(*). Then the subprogram returns to the calling
C program unit. Typically one could do the following
C steps:
C
C step 1. Place the partials of the i-th constraint
C function with respect to variable j in the
C array FJ(i,j), i=1,...,MCON, j=1,...,NVARS.
C step 2. Place the values of the i-th constraint
C equation into FJ(i,NVARS+1).
C step 3. Place the partials of the i-th least squares
C equation with respect to variable j in the
C array FJ(MCON+i,j), i=1,...,MEQUA,
C j=1,...,NVARS.
C step 4. Place the value of the i-th least squares
C equation into FJ(MCON+i,NVARS+1).
C step 5. Return to the calling program unit.
DOUBLE PRECISION FJ(LDFJ,*),X(*),ROPT(*)
DOUBLE PRECISION T(5),F(5)
INTEGER IOPT(*)
DATA T/0.05,0.10,0.40,0.50,1.00/
DATA F/2.206D+00,1.994D+00,1.350D+00,1.216D+00,.7358D0/
DATA MCON,MEQUA,NVARS/1,5,4/
C
C Define the derivatives of the constraint with respect to the x(j).
FJ(1,1) = 0.0D0

FJ(1,2) = 1.D0
FJ(1,3) = 0.D0
FJ(1,4) = -1.D0

C Define the value of this constraint.
FJ(1,5) = X(2) - X(4)

C Define the derivatives and residuals for the data model.
DO 10 I = 1,MEQUA
   E1 = EXP(X(2)*T(I))
   E2 = EXP(X(4)*T(I))
   FJ(MCON+I,1) = E1
   FJ(MCON+I,2) = X(1)*T(I)*E1
   FJ(MCON+I,3) = E2
   FJ(MCON+I,4) = X(3)*T(I)*E2
   FJ(MCON+I,5) = X(1)*E1 + X(3)*E2 - F(I)
10 CONTINUE
RETURN
END

Output from Example 1 Program
-------- ---- --------- -------
MODEL IS H(T) = X(1)*EXP(-T*X(2)) + X(3)*EXP(T*X(4))
X(1),X(2),X(3),X(4) =
   1.999475  -.999801  .500057  -9.953988
RESIDUAL AFTER THE FIT = 4.2408D-04
OUTPUT FLAG FROM SOLVER = 4

Example 2 Using Reverse Communication
-------- ----- ------- ------------
PROGRAM EX02

C Illustrate the use of the Hanson-Krogh nonlinear least
C squares solver for fitting two exponentials to data.
C
C The problem is to find the four variables x(1),...,x(4)
C that are in the model function
C
C h(t) = x(1)*exp(x(2)*t) + x(3)*exp(x(4)*t)
C
C There are values of h(t) given at five values of t,
C t=0.05, 0.1, 0.4, 0.5, and 1.0.
C We also have problem constraints that x(2), x(4) .le. 0, x(1),
C x(3) .ge. 0, and a minimal separation of 0.05 between x(2) and
C x(4). Nothing more about the values of the parameters is known
C except that x(2),x(4) are approximately .ge. 1/min t.
C Thus we have no further knowledge of their values.
C For that reason all of the initial values are set to zero.
C
C Dimension for the nonlinear solver.
DOUBLE PRECISION FJ(6,5),BL(5),BU(5),X(4),ROPT(001),WA(640)

C EDIT on 950228-1300:
DOUBLE PRECISION RNORM
INTEGER IND(5),IOPT(24),IWA(084)
DOUBLE PRECISION T(5),F(5)
EXTERNAL PDA_DQEDEV

DATA LDFJ,LWA,LIWA/6,640,084/

DATA T/0.05,0.10,0.40,0.50,1.00/
DATA F/2.206D+00,1.994D+00,1.350D+00,1.216D+00,.7358D0/

MCON = 1
MEQUA = 5
NVARS = 4

C Define the constraints for variables.
BL(1) = 0.
BL(2) = -25.
BU(2) = 0.
BL(3) = 0.
BL(4) = -25.
BU(4) = 0.

C Define the constraining value (separation) for the arguments.
BL(5) = 0.05

C Define all of the constraint indicators.
IND(1) = 1
IND(2) = 3
IND(3) = 1
IND(4) = 3
IND(5) = 1

C Define the initial values of the variables.
C We don’t know anything at all, so all variables are set zero.
DO 10 J = 1,NVARS
  X(J) = 0.D0
10 CONTINUE

C Tell how much storage we gave the solver.
IWA(1) = LWA
IWA(2) = LIWA
NITERS = 0

C USE REVERSE COMMUNICATION TO EVALUATE THE DERIVATIVES.
IOPT(01)=16
IOPT(02)=1

C NO MORE OPTIONS.
IOPT(03) = 99

20 CONTINUE
CALL PDA_DQED(PDA_DQEDEV,MEQUA,NVARS,MCON,IND,BL,BU,X,FJ,LDFJ,RNORM,
  .IGO,IOPT, ROPT,IWA,WA)
IF (IGO.GT.1) GO TO 40

C COUNT FUNCTION EVALUATIONS.
NITERS = NITERS + 1

C DEFINE THE DERIVATIVES OF THE CONSTRAINT WITH RESPECT TO THE X(J).
FJ(1,1) = 0.D0
FJ(1,2) = 1.D0
FJ(1,3) = 0.D0
FJ(1,4) = -1.D0

C DEFINE THE VALUE OF THIS CONSTRAINT.
FJ(1,5) = X(2) - X(4)
C DEFINE THE DERIVATIVES AND RESIDUALS FOR THE DATA MODEL.
DO 30 I = 1,MEQUA
   E1 = EXP(X(2)*T(I))
   E2 = EXP(X(4)*T(I))
   FJ(MCON+I,1) = E1
   FJ(MCON+I,2) = X(1)*T(I)*E1
   FJ(MCON+I,3) = E2
   FJ(MCON+I,4) = X(3)*T(I)*E2
   FJ(MCON+I,5) = X(1)*E1 + X(3)*E2 - F(I)
30 CONTINUE
GO TO 20
40 CONTINUE
NOUT = 6
WRITE (NOUT,9001) (X(J),J=1,NVARS)
WRITE (NOUT,9011) RNORM
WRITE (NOUT,9021) NITERS, IGO
9001 FORMAT (' MODEL IS H(T) = X(1)*EXP(-T*X(2)) + X(3)*EXP(T*X(4))',/,
            ' X(1),X(2),X(3),X(4) = ',/4F12.6)
9011 FORMAT (' RESIDUAL AFTER THE FIT = ',1PD12.4)
9021 FORMAT (' NUMBER OF EVALUATIONS OF PARAMETER MODEL =',I6,/,,
            ' OUTPUT FLAG FROM SOLVER =',17X,I6)
STOP
END
Output from Example 2 Program
MODEL IS H(T) = X(1)*EXP(-T*X(2)) + X(3)*EXP(T*X(4))
X(1),X(2),X(3),X(4) =
   1.999475 -.999801 .500057 -9.953988
RESIDUAL AFTER THE FIT = 4.2408D-04
NUMBER OF EVALUATIONS OF PARAMETER MODEL = 14
OUTPUT FLAG FROM SOLVER = 4

4. Error Messages for PDA_DQED()
-------------------------------------
'DQED. VALUE OF MEQUA=NO. OF EQUAS. MUST .GT.0. NOW = (I1).'
NERR = 01
IGO=9

'DQED. VALUE OF NVARS=NO. OF EQUAS. MUST .GT.0. NOW = (I1).'
NERR = 02
IGO=10

'DQED. VALUE OF MCON=NO. OF EQUAS. MUST .GE.0. NOW = (I1).'
NERR = 03
IGO=11

'DQED. INVALID OPTION PROCESSED. I1=IOPT(*) ENTRY. I2=IOPT(I1).'
NERR = 04
IGO=12
'DQED. WA(*) STORAGE SHORT. I1=AMOUNT NEEDED. I2=AMOUNT GIVEN.'
NERR = 05
IGO=13

'DQED. IWA(*) STORAGE SHORT. I1=AMOUNT NEEDED. I2=AMOUNT GIVEN.'
NERR = 06
IGO=14

'DQEDMN. INVALID OPTION PROCESSED. I1=IOPT(*) ENTRY.  I2=IOPT(I1).'
NERR=07
IGO=15

'DQEDIP. INVALID OPTION PROCESSED. I1=IOPT(*) ENTRY.  I2=IOPT(I1).'
NERR=08
IGO=16

'DQED. THE EVALUATOR PROGRAM DQEDEV MUST BE WRITTEN BY THE USER.'
NERR=09
IGO=17

'DQED. BOUND INDICATORS MUST BE 1-4. NOW I1=J, I2=IND(I1).'
NERR=10
IGO=18

5. References
-----------
***REFERENCES
Dongarra, J. J., Bunch, J. R., Moler, C. B, Stewart, G. W.,
PA, (1979).

Hanson, R. J., "Least Squares with Bounds and Linear

p. 815-843.

***END PROLOGUE _DA_DQED
REVISED 870204-1100
REVISED 970224-1230
Name changed to PDA_DQED from DQED.
REVISED YYMMD HHMM
PDA_DR2NAG
Convert FFTPACK Hermitian Fourier transform array into equivalent NAG array

Description:
c.f. PDA_R2NAG.
PDA_DRFFTB
Backward transform of a real coefficient array.

Description:
c.f. [PDA_RFFTB.]
PDA_DRFFTF
Forward transform of a real periodic sequence.

Description:
c.f. [PDA_RFFTF].
PDA_DRFFTI
Initialize PDA_DRFFTF and PDA_DRFFTB.

Description:
c.f. PDA_RFFTI
PDA_DSORT
Sort array and optionally make same interchanges in auxiliary array.

Origin:
SLATEC / CAMSUN

Implementation Status:
The routine will now return an error status as supplied by PDA_XERMSG.
SUBROUTINE PDA_DSORT (DX, DY, N, KFLAG, STATUS)

***BEGIN PROLOGUE PDA_DSORT
***PURPOSE Sort an array and optionally make the same interchanges in an auxiliary array. The array may be sorted in increasing or decreasing order. A slightly modified QUICKSORT algorithm is used.
***LIBRARY SLATEC
***CATEGORY N6A2B
***TYPE DOUBLE PRECISION (SSORT-S, PDA_DSORT-D, ISORT-I)
***KEYWORDS SINGLETON QUICKSORT, SORT, SORTING
***AUTHOR Jones, R. E., (SNLA)
Wisniewski, J. A., (SNLA)
***DESCRIPTION
PDA_DSORT sorts array DX and optionally makes the same interchanges in array DY. The array DX may be sorted in increasing order or decreasing order. A slightly modified quicksort algorithm is used.

Description of Parameters
DX - array of values to be sorted (usually abscissas)
DY - array to be (optionally) carried along
N - number of values in array DX to be sorted
KFLAG - control parameter
= 2 means sort DX in increasing order and carry DY along.
= 1 means sort DX in increasing order (ignoring DY)
= -1 means sort DX in decreasing order (ignoring DY)
= -2 means sort DX in decreasing order and carry DY along.
STATUS - Returned error status.
The status must be zero on entry. This routine does not check the status on entry.

***ROUTINES CALLED PDA_XERMSG
***REVISION HISTORY (YYMMDD)
761101 DATE WRITTEN
761118 Modified to use the Singleton quicksort algorithm. (JAW)
890531 Changed all specific intrinsics to generic. (WRB)
890831 Modified array declarations. (WRB)
891009 Removed unreferenced statement labels. (WRB)
891024 Changed category. (WRB)
891024 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900315 CALLs to XERROR changed to CALLs to PDA_XERMSG. (THJ)
911012 Declared all variables; changed X,Y to DX,DY; changed code to parallel SSORT. (M. McClain)
920501 Reformatted the REFERENCES section. (DWL, WRB)
920519 Clarified error messages. (DWL)
920801 Declarations section rebuilt and code restructured to use IF-THEN-ELSE-ENDIF. (RWC, WRB)
950403 Implement status. (HME)
***END PROLOGUE   PDA_DSORT
PDA_I1MACH
Integer machine dependent constants

Origin:
SLATEC / CAMSUN

Implementation Status:
Separate versions for ‘alpha_OSF1’ and ‘sun4_Solaris’ exist.
INTEGER FUNCTION PDA_I1MACH (I)

***BEGIN PROLOGUE PDA_I1MACH
***PURPOSE Return integer machine dependent constants.
***LIBRARY SLATEC
***CATEGORY R1
***TYPE INTEGER (PDA_I1MACH-I)
***KEYWORDS MACHINE CONSTANTS
***AUTHOR Fox, P. A., (Bell Labs)
       Hall, A. D., (Bell Labs)
       Schryer, N. L., (Bell Labs)
***DESCRIPTION

PDA_I1MACH can be used to obtain machine-dependent parameters for the
local machine environment. It is a function subprogram with one
(input) argument and can be referenced as follows:

\[ K = \text{PDA}_I1\text{MACH}(I) \]

where \( I=1, \ldots, 16 \). The (output) value of \( K \) above is determined by
the (input) value of \( I \). The results for various values of \( I \) are
discussed below.

I/O unit numbers:
PDA_I1MACH(1) = the standard input unit.
PDA_I1MACH(2) = the standard output unit.
PDA_I1MACH(3) = the standard punch unit.
PDA_I1MACH(4) = the standard error message unit.

Words:
PDA_I1MACH(5) = the number of bits per integer storage unit.
PDA_I1MACH(6) = the number of characters per integer storage unit.

Integers:
assume integers are represented in the S-digit, base-A form

\[ \text{sign} \left( X(S-1)A^{S-1} + \ldots + X(1)A + X(0) \right) \]

where \( 0 \leq X(I) < A \) for \( I=0, \ldots, S-1 \).
PDA_I1MACH(7) = A, the base.
PDA_I1MACH(8) = S, the number of base-A digits.
PDA_I1MACH(9) = A**S - 1, the largest magnitude.

Floating-Point Numbers:
Assume floating-point numbers are represented in the T-digit,
base-B form

\[ \text{sign} \left( B^{*E} \left( X(1)/B + \ldots + X(T)/B^{*B} \right) \right) \]

where \( 0 \leq X(I) < B \) for \( I=1, \ldots, T \),
\( 0 \leq X(1) \), and \( \text{EMIN} \leq E \leq \text{EMAX} \).
PDA_I1MACH(10) = B, the base.

Single-Precision:
PDA_I1MACH(11) = T, the number of base-B digits.
PDA_I1MACH(12) = EMIN, the smallest exponent E.
PDA_I1MACH(13) = EMAX, the largest exponent E.

Double-Precision:
PDA_I1MACH(14) = T, the number of base-B digits.
PDA_I1MACH(15) = EMIN, the smallest exponent E.
PDA_I1MACH(16) = EMAX, the largest exponent E.

To alter this function for a particular environment, the desired set of DATA statements should be activated by removing the C from column 1. Also, the values of PDA_I1MACH(1) - PDA_I1MACH(4) should be checked for consistency with the local operating system.


***ROUTINES CALLED (NONE)

***REVISION HISTORY (YYMMDD)

750101 DATE WRITTEN
891012 Added VAX G-floating constants. (WRB)
891012 REVISION DATE from Version 3.2
891214 Prologue converted to Version 4.0 format. (BAB)
900618 Added DEC RISC constants. (WRB)
900723 Added IBM RS 6000 constants. (WRB)
901009 Correct PDA_I1MACH(7) for IBM Mainframes. Should be 2 not 16. (RWC)
910710 Added HP 730 constants. (SMR)
911114 Added Convex IEEE constants. (WRB)
920121 Added SUN -r8 compiler option constants. (WRB)
920229 Added Touchstone Delta i860 constants. (WRB)
920501 Reformatted the REFERENCES section. (WRB)
920625 Added Convex -p8 and -pd8 compiler option constants. (BKS, WRB)
930201 Added DEC Alpha and SGI constants. (RWC and WRB)
930618 Corrected PDA_I1MACH(5) for Convex -p8 and -pd8 compiler options. (DWL, RWC and WRB).
950404 If index out of range, return value zero, but return. (HME).

***END PROLOGUE PDA_I1MACH
PDA_IDBVIP
Performs 2-D bivariate interpolation when the data is irregularly scattered in the x-y plane.

Origin:
TOMS/NETLIB

Implementation Status:
The warning messages are no longer printed. The same information is returned in the argument ISTAT.
c The subroutine performs bivariate interpolation when the projections of the data points in the x-y plane are irregularly distributed in the plane.

c The input parameters are

- md = mode of computation (must be 1, 2, or 3),
  - 1 for new ncp and/or new xd-yd,
  - 2 for old ncp, old xd-yd, new xi-yi,
  - 3 for old ncp, old xd-yd, old xi-yi,
- ncp = number of additional data points used for estimating partial derivatives at each data point (must be 2 or greater, but smaller than ndp),
- ndp = number of data points (must be 4 or greater),
- xd = array of dimension ndp containing the x coordinates of the data points,
- yd = array of dimension ndp containing the y coordinates of the data points,
- zd = array of dimension ndp containing the z coordinates of the data points,
- nip = number of output points at which interpolation is to be performed (must be 1 or greater),
- xi = array of dimension nip containing the x coordinates of the output points,
- yi = array of dimension nip containing the y coordinates of the output points.

c The output parameter is

- zi = array of dimension nip where interpolated z values are to be stored.
- istat = error message.
- status = Starlink error status

c The other parameters are

- iwk = integer array of dimension max0(31,27+ncp)*ndp+nip used internally as a work area,
- wk = array of dimension 8*ndp used internally as a work area.

c The very first call to this subroutine and the call with a new ncp value, a new ndp value, and/or new contents of the xd and yd arrays must be made with md=1. The call with md=2 must be preceded by another call with the same ncp and ndp values and with the same contents of the xd and yd arrays. The call with md=3 must be preceded by another call with the same ncp, ndp, c nip values and with the same contents of the xd, yd, xi, c and yi arrays. Between the call with md=2 or md=3 and its preceeding call, the iwk and wk arrays must not be disturbed. c use of a value between 3 and 5 (inclusive) for ncp is recom- c mended unless there are evidences that dictate otherwise.

c This subroutine calls the idcldp, idlctn, idpdrv, idptip, and idtang subroutines.
PDA_IDSFFT

Performs smooth surface fitting when the projections of the data points in the x-y plane are irregularly distributed in the plane.

Origin:
TOMS/NETLIB

Implementation Status:
The warning messages are no longer printed. The same information is returned in the argument ISTAT.
this subroutine performs smooth surface fitting when the projections of the data points in the x-y plane are irregularly distributed in the plane.

the input parameters are

- \( \text{md} \) = mode of computation (must be 1, 2, or 3),
  - 1 for new \( ncp \) and/or new \( xd-yd \),
  - 2 for old \( ncp \), old \( xd-yd \), new \( xi-yi \),
  - 3 for old \( ncp \), old \( xd-yd \), old \( xi-yi \),
- \( ncp \) = number of additional data points used for estimating partial derivatives at each data point (must be 2 or greater, but smaller than \( ndp \)),
- \( ndp \) = number of data points (must be 4 or greater),
- \( xd \) = array of dimension \( ndp \) containing the x coordinates of the data points,
- \( yd \) = array of dimension \( ndp \) containing the y coordinates of the data points,
- \( zd \) = array of dimension \( ndp \) containing the z coordinates of the data points,
- \( nxi \) = number of output grid points in the x coordinate (must be 1 or greater),
- \( nyi \) = number of output grid points in the y coordinate (must be 1 or greater),
- \( xi \) = array of dimension \( nxi \) containing the x coordinates of the output grid points,
- \( yi \) = array of dimension \( nyi \) containing the y coordinates of the output grid points.

the output parameter is

- \( zi \) = doubly-dimensioned array of dimension \((nxi,nyi)\),
  - where the interpolated z values at the output grid points are to be stored.
- \( istat \) = error message.
- \( status \) = Starlink error status

the other parameters are

- \( iwk \) = integer array of dimension
  - \( \text{max}(31,27+ncp)\times ndp+nxi\times nyi \)
  - used internally as a work area,
- \( wk \) = array of dimension \( 5\times ndp \) used internally as a work area.

the very first call to this subroutine and the call with a new ncp value, a new ndp value, and/or new contents of the \( xd \) and \( yd \) arrays must be made with \( md=1 \). the call with \( md=2 \) must be preceded by another call with the same ncp and ndp values and with the same contents of the \( xd \) and \( yd \) arrays. the call with \( md=3 \) must be preceded by another call with the same ncp, ndp, \( nxi \), and \( nyi \) values and with the same contents of the \( xd \), \( yd \), \( xi \), and \( yi \) arrays. between the call with \( md=2 \) or \( md=3 \) and its preceding call, the \( iwk \) and \( wk \) arrays must not be disturbed.

use of a value between 3 and 5 (inclusive) for ncp is recommended unless there are evidences that dictate otherwise.
c this subroutine calls the idcldp, idgrid, idpdrv, idptip, and
  c idtang subroutines.
Description:
This routine inverts a permutation in place. It can be used to transform an index vector (from a sort) into a rank vector and vice versa.

Invocation:
CALL PDA_IPERM( N, X )

Arguments:

N = INTEGER (Read)
Number of elements.

X( N ) = _INTEGER (Read and Write)
The permutation. On exit this contains the inverse.

Notes:
The permutation must consist of positive integers. The permutation inverse Y(X(I))=I for I=1,N can be formed trivially with 2*N arrays.

References :
The Art of Computer Programming, Fundamental Algorithms Vol 1, by Donald E. Knuth (Addison-Wesley).

Timing :
Proportional to N.
PDA_LMDIF
Minimise the sum of the squares of m nonlinear functions in n variables, function only.

Origin:
MINPACK / NETLIB
**subroutine pda_lmdif**

The purpose of 

### subroutine pda_lmdif

The purpose of pda_lmdif is to minimize the sum of the squares of m nonlinear functions in n variables by a modification of the levenberg-marquardt algorithm. The user must provide a subroutine which calculates the functions. The jacobian is then calculated by a forward-difference approximation.

The subroutine statement is

```plaintext
subroutine pda_lmdif(fcn,m,n,x,fvec,ftol,xtol,gtol,maxfev,epsfcn,
   diag,mode,factor,nprint,info,nfev,fjac,ldfjac,
   ipvt,qtf,wa1,wa2,wa3,wa4)
```

Where

- **fcn** is the name of the user-supplied subroutine which calculates the functions. Fcn must be declared in an external statement in the user calling program, and should be written as follows.

  ```plaintext
  subroutine fcn(m,n,x,fvec,iflag)
  integer m,n,iflag
  double precision x(n),fvec(m)
  
  calculate the functions at x and return this vector in fvec.
  
  return
  end
  ```

  The value of iflag should not be changed by fcn unless the user wants to terminate execution of pda_lmdif. In this case set iflag to a negative integer.

- **m** is a positive integer input variable set to the number of functions.
- **n** is a positive integer input variable set to the number of variables. n must not exceed m.
- **x** is an array of length n. On input x must contain an initial estimate of the solution vector. On output x contains the final estimate of the solution vector.
- **fvec** is an output array of length m which contains the functions evaluated at the output x.
ftol is a nonnegative input variable. termination occurs when both the actual and predicted relative reductions in the sum of squares are at most ftol. therefore, ftol measures the relative error desired in the sum of squares.

xtol is a nonnegative input variable. termination occurs when the relative error between two consecutive iterates is at most xtol. therefore, xtol measures the relative error desired in the approximate solution.

gtol is a nonnegative input variable. termination occurs when the cosine of the angle between fvec and any column of the jacobian is at most gtol in absolute value. therefore, gtol measures the orthogonality desired between the function vector and the columns of the jacobian.

maxfev is a positive integer input variable. termination occurs when the number of calls to fcn is at least maxfev by the end of an iteration.

epsfcn is an input variable used in determining a suitable step length for the forward-difference approximation. this approximation assumes that the relative errors in the functions are of the order of epsfcn. if epsfcn is less than the machine precision, it is assumed that the relative errors in the functions are of the order of the machine precision.

diag is an array of length n. if mode = 1 (see below), diag is internally set. if mode = 2, diag must contain positive entries that serve as multiplicative scale factors for the variables.

mode is an integer input variable. if mode = 1, the variables will be scaled internally. if mode = 2, the scaling is specified by the input diag. other values of mode are equivalent to mode = 1.

factor is a positive input variable used in determining the initial step bound. this bound is set to the product of factor and the euclidean norm of diag*x if nonzero, or else to factor itself. in most cases factor should lie in the interval (.1,100.). 100. is a generally recommended value.

nprint is an integer input variable that enables controlled printing of iterates if it is positive. in this case, fcn is called with iflag = 0 at the beginning of the first iteration and every nprint iterations thereafter and immediately prior to return, with x and fvec available for printing. if nprint is not positive, no special calls of fcn with iflag = 0 are made.
info is an integer output variable. If the user has terminated execution, info is set to the (negative) value of iflag. See description of fcn. Otherwise, info is set as follows.

- info = 0 improper input parameters.
- info = 1 both actual and predicted relative reductions in the sum of squares are at most ftol.
- info = 2 relative error between two consecutive iterates is at most xtol.
- info = 3 conditions for info = 1 and info = 2 both hold.
- info = 4 the cosine of the angle between fvec and any column of the jacobian is at most gtol in absolute value.
- info = 5 number of calls to fcn has reached or exceeded maxfev.
- info = 6 ftol is too small. No further reduction in the sum of squares is possible.
- info = 7 xtol is too small. No further improvement in the approximate solution x is possible.
- info = 8 gtol is too small. fvec is orthogonal to the columns of the jacobian to machine precision.

nfev is an integer output variable set to the number of calls to fcn.

fjac is an output m by n array. The upper n by n submatrix of fjac contains an upper triangular matrix r with diagonal elements of nonincreasing magnitude such that

\[ p \,(\text{jac} \,*\text{jac})\,p = r \,*r, \]

where p is a permutation matrix and jac is the final calculated jacobian. Column j of p is column ipvt(j) (see below) of the identity matrix. The lower trapezoidal part of fjac contains information generated during the computation of r.

ldfjac is a positive integer input variable not less than m which specifies the leading dimension of the array fjac.

ipvt is an integer output array of length n. ipvt defines a permutation matrix p such that jac*p = q*r, where jac is the final calculated jacobian, q is
orthogonal (not stored), and \( r \) is upper triangular with diagonal elements of nonincreasing magnitude. column \( j \) of \( p \) is column \( \text{ipvt}(j) \) of the identity matrix.

\[ qt'f \] is an output array of length \( n \) which contains the first \( n \) elements of the vector \( (q \cdot q^T) \cdot fvec \).

\( w_1, w_2, \) and \( w_3 \) are work arrays of length \( n \).

\( w_4 \) is a work array of length \( m \).

subprograms called
user-supplied ...... \( fcn \)

minpack-supplied ... \( \text{pda_dpmpar}, \text{pda_enorm}, \text{pda_fdjac2}, \text{pda_lmpar}, \text{pda_qrfac} \)

fortran-supplied ... \( \text{dabs}, \text{dmax1}, \text{dmin1}, \text{dsqrt}, \text{mod} \)

argonne national laboratory. minpack project. march 1980.
burton s. garbow, kenneth e. hillstrom, jorge j. more

**********
PDA_LMDIF1
Minimise the sum of the squares of m nonlinear functions in n variables, simple interface to PDA_LMDIF.

Origin:
MINPACK / NETLIB
subroutine pda_lmdif1(fcn,m,n,x,fvec,tol,info,iwa,wa,lwa)

**********
subroutine pda_lmdif1

the purpose of pda_lmdif1 is to minimize the sum of the squares of 
m nonlinear functions in n variables by a modification of the 
levenberg-marquardt algorithm. this is done by using the more 
general least-squares solver pda_lmdif. the user must provide a 
subroutine which calculates the functions. the jacobian is 
then calculated by a forward-difference approximation.

the subroutine statement is

subroutine pda_lmdif1(fcn,m,n,x,fvec,tol,info,iwa,wa,lwa)

where

fcn is the name of the user-supplied subroutine which 
calculates the functions. fcn must be declared 
in an external statement in the user calling 
program, and should be written as follows.

subroutine fcn(m,n,x,fvec,iflag)
integer m,n,iflag
double precision x(n),fvec(m)
----------
calculate the functions at x and
return this vector in fvec.
----------
return
end

the value of iflag should not be changed by fcn unless 
the user wants to terminate execution of pda_lmdif1. 
in this case set iflag to a negative integer.

m is a positive integer input variable set to the number 
of functions.

n is a positive integer input variable set to the number 
of variables. n must not exceed m.

x is an array of length n. on input x must contain 
an initial estimate of the solution vector. on output x 
contains the final estimate of the solution vector.

fvec is an output array of length m which contains 
the functions evaluated at the output x.

tol is a nonnegative input variable. termination occurs 
when the algorithm estimates either that the relative
error in the sum of squares is at most tol or that
the relative error between x and the solution is at
most tol.

info is an integer output variable. if the user has
terminated execution, info is set to the (negative)
value of iflag. see description of fcn. otherwise,
info is set as follows.

info = 0 improper input parameters.

info = 1 algorithm estimates that the relative error
in the sum of squares is at most tol.

info = 2 algorithm estimates that the relative error
between x and the solution is at most tol.

info = 3 conditions for info = 1 and info = 2 both hold.

info = 4 fvec is orthogonal to the columns of the
jacobian to machine precision.

info = 5 number of calls to fcn has reached or
exceeded 200*(n+1).

info = 6 tol is too small. no further reduction in
the sum of squares is possible.

info = 7 tol is too small. no further improvement in
the approximate solution x is possible.

iwa is an integer work array of length n.

wa is a work array of length lwa.

lwa is a positive integer input variable not less than
m*n+5*n+m.

subprograms called

user-supplied ...... fcn

minpack-supplied ... pda_lmdif

argonne national laboratory. minpack project. march 1980.
burton s. garbow, kenneth e. hillstrom, jorge j. more

**********
PDA_LSQR
Solves sparse unsymmetric, linear least squares and damped least squares problems

Origin:
NETLIB
SUBROUTINE PDA_LSQR ( M, N, APROD, DAMP, LENIW, LENRW, IW, RW, 
: U, V, W, X, SE, ATOL, BTOL, CONLIM, ITNLIM, 
: ISTOP, ITN, ANORM, ACOND, RNORM, ARNORM, 
: XNORM )

EXTERNAL APROD
INTEGER M, N, LENIW, LENRW, ITNLIM, ISTOP, ITN
INTEGER IW(LENIW)
DOUBLE PRECISION RW(LENRW), U(M), V(N), W(N), X(N), SE(N), 
: ATOL, BTOL, CONLIM, DAMP, 
: ANORM, ACOND, RNORM, ARNORM, XNORM

-----------------------------------------------------------------------

PDA_LSQR finds a solution x to the following problems:

1. Unsymmetric equations -- solve \( A \times x = b \)

2. Linear least squares -- solve \( A \times x = b \)
   in the least-squares sense

3. Damped least squares -- solve \( ( A \times + \text{damp} \times I ) \times x = ( b ) \)
   \( ( 0 ) \)
   in the least-squares sense

where \( A \) is a matrix with \( m \) rows and \( n \) columns, \( b \) is an
\( m \)-vector, and \( \text{damp} \) is a scalar. (All quantities are real.)

The matrix \( A \) is intended to be large and sparse. It is accessed
by means of subroutine calls of the form

\[ \text{CALL} \ APROD ( \text{mode}, m, n, x, y, \text{LENIW}, \text{LENRW}, \text{IW}, \text{RW} ) \]

which must perform the following functions:

If MODE = 1, compute \( y = y + A \times x \).
If MODE = 2, compute \( x = x + A(\text{transpose}) \times y \).

The vectors \( x \) and \( y \) are input parameters in both cases.
If mode = 1, \( y \) should be altered without changing \( x \).
If mode = 2, \( x \) should be altered without changing \( y \).
The parameters \( \text{LENIW}, \text{LENRW}, \text{IW}, \text{RW} \) may be used for workspace
as described below.

The rhs vector \( b \) is input via \( U \), and subsequently overwritten.

Note: PDA_LSQR uses an iterative method to approximate the solution.
The number of iterations required to reach a certain accuracy
depends strongly on the scaling of the problem. Poor scaling of
the rows or columns of \( A \) should therefore be avoided where
possible.

For example, in problem 1 the solution is unaltered by
row-scaling. If a row of \( A \) is very small or large compared to
the other rows of \( A \), the corresponding row of \( ( A \ b ) \) should be
scaled up or down.

In problems 1 and 2, the solution \( x \) is easily recovered following column-scaling. Unless better information is known, the nonzero columns of \( A \) should be scaled so that they all have the same Euclidean norm (e.g., 1.0).

In problem 3, there is no freedom to re-scale if \( \text{damp} \) is nonzero. However, the value of \( \text{damp} \) should be assigned only after attention has been paid to the scaling of \( A \).

The parameter \( \text{damp} \) is intended to help regularize ill-conditioned systems, by preventing the true solution from being very large. Another aid to regularization is provided by the parameter \( \text{ACOND} \), which may be used to terminate iterations before the computed solution becomes very large.

Notation
--------

The following quantities are used in discussing the subroutine parameters:

\[
\begin{align*}
A_{\text{bar}} &= \begin{pmatrix} A \\ \text{damp}I \end{pmatrix}, & b_{\text{bar}} &= \begin{pmatrix} b \\ 0 \end{pmatrix} \\
\mathbf{r} &= b - A\mathbf{x}, & \mathbf{r}_{\text{bar}} &= b_{\text{bar}} - A_{\text{bar}}\mathbf{x} \\
\text{rnorm} &= \sqrt{\|\mathbf{r}\|^2 + \text{damp}^2 \|\mathbf{x}\|^2} \\
\text{RELPR} &= \text{the relative precision of floating-point arithmetic on the machine being used. For example, on the IBM 370, RELPR is about 1.0E-6 and 1.0D-16 in single and double precision respectively.}
\end{align*}
\]

\( \text{PDA\_LSQR} \) minimizes the function \( \text{rnorm} \) with respect to \( \mathbf{x} \).

Parameters
---------

\( M \) input \( m \), the number of rows in \( A \).

\( N \) input \( n \), the number of columns in \( A \).

\( \text{APROD} \) external See above.

\( \text{DAMP} \) input The damping parameter for problem 3 above. (\( \text{DAMP} \) should be 0.0 for problems 1 and 2.) If the system \( A\mathbf{x} = b \) is incompatible, values of \( \text{DAMP} \) in the range 0 to \( \text{sqrt}(\text{RELPR})\|\mathbf{A}\| \) will probably have a negligible effect.
Larger values of DAMP will tend to decrease the norm of \( \mathbf{x} \) and reduce the number of iterations required by PDA_LSQR.

The work per iteration and the storage needed by PDA_LSQR are the same for all values of DAMP.

- **LENIW** input: The length of the workspace array IW.
- **LENRW** input: The length of the workspace array RW.
- **IW** workspace: An integer array of length LENIW.
- **RW** workspace: A real array of length LENRW.

Note: PDA_LSQR does not explicitly use the previous four parameters, but passes them to subroutine APROD for possible use as workspace. If APROD does not need IW or RW, the values LENIW = 1 or LENRW = 1 should be used, and the actual parameters corresponding to IW or RW may be any convenient array of suitable type.

- **U(M)** input: The rhs vector \( \mathbf{b} \). Beware that U is over-written by PDA_LSQR.
- **V(N)** workspace
- **W(N)** workspace
- **X(N)** output: Returns the computed solution \( \mathbf{x} \).
- **SE(N)** output: Returns standard error estimates for the components of \( \mathbf{X} \). For each \( i \), \( SE(i) \) is set to the value \( \text{rnorm} \times \sqrt{\text{sigma}(i,i) / T} \), where \( \text{sigma}(i,i) \) is an estimate of the \( i \)-th diagonal of the inverse of Abar(transpose)*Abar and
  - \( T = 1 \) if \( m \leq n \),
  - \( T = m - n \) if \( m > n \) and DAMP = 0,
  - \( T = m \) if DAMP .NE. 0.

- **ATOL** input: An estimate of the relative error in the data defining the matrix A. For example, if A is accurate to about 6 digits, set ATOL = 1.0E-6.

- **BTOL** input: An estimate of the relative error in the data defining the rhs vector \( \mathbf{b} \). For example, if \( \mathbf{b} \) is accurate to about 6 digits, set BTOL = 1.0E-6.

- **CONLIM** input: An upper limit on \( \text{cond}(\text{Abar}) \), the apparent condition number of the matrix Abar. Iterations will be terminated if a computed estimate of \( \text{cond}(\text{Abar}) \) exceeds CONLIM. This is intended to prevent certain small or zero singular values of A or Abar from coming into effect and causing unwanted growth in the computed solution.
CONLIM and DAMP may be used separately or together to regularize ill-conditioned systems.

Normally, CONLIM should be in the range 1000 to 1/RELPR.
Suggested value:
CONLIM = 1/(100*RELPR) for compatible systems,
CONLIM = 1/(10*sqrt(RELPR)) for least squares.

Note: If the user is not concerned about the parameters ATOL, BTOL and CONLIM, any or all of them may be set to zero. The effect will be the same as the values RELPR, RELPR and 1/RELPR respectively.

ITNLIM input An upper limit on the number of iterations.
Suggested value:
ITNLIM = n/2 for well-conditioned systems
with clustered singular values,
ITNLIM = 4*n otherwise.

ISTOP output An integer giving the reason for termination:

0  x = 0 is the exact solution.
   No iterations were performed.

1  The equations A*x = b are probably compatible. Norm(A*x - b) is sufficiently small, given the values of ATOL and BTOL.

2  The system A*x = b is probably not compatible. A least-squares solution has been obtained that is sufficiently accurate, given the value of ATOL.

3  An estimate of cond(Abar) has exceeded CONLIM. The system A*x = b appears to be ill-conditioned. Otherwise, there could be an error in subroutine APROD.

4  The equations A*x = b are probably compatible. Norm(A*x - b) is as small as seems reasonable on this machine.

5  The system A*x = b is probably not compatible. A least-squares solution has been obtained that is as accurate as seems reasonable on this machine.

6  Cond(Abar) seems to be so large that there is no point in doing further iterations, given the precision of this machine. There could be an error in subroutine APROD.
The iteration limit ITNLIM was reached.

ITN output The number of iterations performed.

ANORM output An estimate of the Frobenius norm of Abar. This is the square-root of the sum of squares of the elements of Abar. If DAMP is small and if the columns of A have all been scaled to have length 1.0, ANORM should increase to roughly sqrt(n). A radically different value for ANORM may indicate an error in subroutine APROD (there may be an inconsistency between modes 1 and 2).

ACOND output An estimate of cond(Abar), the condition number of Abar. A very high value of ACOND may again indicate an error in APROD.

RNORM output An estimate of the final value of norm(rbar), the function being minimized (see notation above). This will be small if A*x = b has a solution.

ARNORM output An estimate of the final value of norm( Abar(transpose)*rbar ), the norm of the residual for the usual normal equations. This should be small in all cases. (ARNORM will often be smaller than the true value computed from the output vector X.)

XNORM output An estimate of the norm of the final solution vector X.

Precision
---------

The number of iterations required by PDA_LSQR will usually decrease if the computation is performed in higher precision. To convert PDA_LSQR between single and double precision, change the words

DOUBLE PRECISION
DCOPY, DNM2, DSCAL

to the appropriate FORTRAN and BLAS equivalents. Also change 'D+' or 'E+' in the PARAMETER statement.

References
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PDA_LSQR development:
22 Feb 1982: LSQR sent to ACM TOMS to become Algorithm 583.
15 Sep 1985: Final F66 version. LSQR sent to "misc" in netlib.
13 Oct 1987: Bug (Robert Davies, DSIR). Have to delete
   IF ( (ONE + DABS(T)) .LE. ONE ) GO TO 200
from loop 200. The test was an attempt to reduce underflows, but caused W(I) not to be updated.
04 May 1989: Bug (David Gay, AT&T). When the second BETA is zero,
   RNORM = 0 and
   TEST2 = ARNORM / (ANORM * RNORM) overflows.
   Fixed by testing for RNORM = 0.
05 May 1989: Sent to "misc" in netlib.
Michael A. Saunders (na.saunders @ NA-net.stanford.edu)
Department of Operations Research
Stanford University
Stanford, CA 94305-4022.
19 Sep 1996: Peter W. Draper. Removed NOUT argument and renamed PDA_LSQR. PDA routines may not write output.
PDA_NAG2C

Convert NAG complex Fourier transform array into array usable by FFTPACK routine PDA_CFFTB

Description:
This subroutine returns a modified version of the supplied Fourier coefficients (as produced by NAG subroutine C06FCF). An inverse FFT can be performed on the returned array using FFTPACK routine PDA_CFFTB, and the resulting inverse will have the same normalisation as the original data transformed using PDA_CFFTF. See PDA_C2NAG for more details.

Invocation:
CALL PDA_NAG2C( NP, X, Y, R )

Arguments:
NP = INTEGER (Given)
The number of points in the transform.
X( NP ) = REAL (Given)
The real coefficients, in NAG format.
Y( NP ) = REAL (Given)
The imaginary coefficients, in NAG format.
R( 2, NP ) = REAL (Returned)
The output coefficients, in FFTPACK format.

Implementation Status:
A double precision version PDA_DNAG2C of the routine exists.

Notes:
A call to PDA_C2NAG followed by a call to PDA_NAG2C will result in the original data being divided by NP.
PDA_NAG2R

Convert NAG Hermitian Fourier transform array into array usable by FFTPACK routine PDA_RFFTB

Description:
This subroutine modifies the supplied array of Fourier coefficients (as produced by NAG subroutine C06FAF) so that an inverse FFT can be performed on them using FFTPACK routine PDA_RFFTB. The resulting inverse will have the same normalisation as the original data transformed using PDA_RFFTF.

Invocation:
CALL PDA_NAG2R( NP, R )

Arguments:
NP = INTEGER (Given)
The size of the array.

R( NP ) = REAL (Given and Returned)
The array holding the Fourier coefficients. Supplied in NAG format and returned in FFTPACK format.

Implementation Status:
A double precision version PDA_DNAG2R of the routine exists.

Notes:
A call to PDA_R2NAG followed by a call to PDA_NAG2R will result in the original data being divided by NP.
Some speed is sacrificed in order to perform the conversion in-situ.
PDA_NFFTB
Backward FFT of N-dimensional complex array

Description:
The supplied Fourier coefficients in X and Y are replaced by the corresponding spatial data obtained by doing an inverse Fourier transform. See the forward FFT routine PDA_NFFTF for more details.

Invocation:
CALL PDA_NFFTB( NDIM, DIM, X, Y, WORK, ISTAT )

Arguments:

NDIM = INTEGER (Given)
The number of dimensions. This should be no more than 20.

DIM( NDIM ) = INTEGER (Given)
The size of each dimension.

X( * ) = REAL (Given and Returned)
Supplied holding the real parts of the Fourier coefficients. Returned holding the real parts of the spatial data. The array should have the number of elements implied by NDIM and DIM.

Y( * ) = REAL (Given and Returned)
Supplied holding the imaginary parts of the Fourier coefficients. Returned holding the imaginary parts of the spatial data. The array should have the number of elements implied by NDIM and DIM.

WORK( * ) = REAL (Given and Returned)
A work array. This should have at least (6*DimMax + 15) elements where DimMax is the maximum of the values supplied in DIM.

ISTAT = INTEGER (Returned)
If the value of NDIM is greater than 20 or less than 1, then ISTAT is returned equal to 1, and the values in X and Y are left unchanged. Otherwise, ISTAT is returned equal to 0.

Implementation Status:
A double precision version PDA_DNFFTB of the routine exists.
PDA_NFFTF
Forward FFT of N-dimensional complex array

Description:
The supplied data values in X and Y are replaced by the coefficients of the Fourier transform of the supplied data. The coefficients are normalised so that a subsequent call to PDA_NFFTB to perform a backward FFT would restore the original data values.
The multi-dimensional FFT is implemented using one-dimensional FFTPACK routines. First each row (i.e. a line of pixels parallel to the first axis) in the supplied array is transformed, the Fourier coefficients replacing the supplied data. Then each column (i.e. a line of pixels parallel to the second axis) is transformed. Then each line of pixels parallel to the third axis is transformed, etc. Each dimension is transformed in this way. Most of the complications in the code come from needing to work in an unknown number of dimensions. Two addressing systems are used for each pixel; 1) the vector (i.e. one-dimensional) index into the supplied arrays, and 2) the corresponding Cartesian pixel indices.

Invocation:
CALL PDA_NFFTF( NDIM, DIM, X, Y, WORK, ISTAT )

Arguments:
NDIM = INTEGER (Given)
The number of dimensions. This should be no more than 20.
DIM( NDIM ) = INTEGER (Given)
The size of each dimension.
X( * ) = REAL (Given and Returned)
Supplied holding the real parts of the complex data values. Returned holding the real parts of the Fourier coefficients. The array should have the number of elements implied by NDIM ande DIM.
Y( * ) = REAL (Given and Returned)
Supplied holding the imaginary parts of the complex data values. Returned holding the imaginary parts of the Fourier coefficients. The array should have the number of elements implied by NDIM ande DIM.
WORK( * ) = REAL (Given and Returned)
A work array. This should have at least ( 6*DimMax + 15 ) elements where DimMax is the maximum of the values supplied in DIM.
ISTAT = INTEGER (Returned)
If the value of NDIM is greater than 20 or less than 1, then ISTAT is returned equal to 1, and the values in X and Y are left unchanged. Otherwise, ISTAT is returned equal to 0.

Implementation Status:
A double precision version PDA_DNFFTF of the routine exists.
PDA_NSCOR
Calculates the approximate expected values of normal order statistics.

Origin:
Applied Statistics / Statlib Archive
SUBROUTINE PDA_NSCOR( S, N, N2, IER )


calculates approximate expected values of normal order statistics.
claimed accuracy is 0.0001, though usually accurate to 5-6 dec.

arguments:
s(n2) = output, the first n2 expected values (double precision).
n = input, the sample size.
n2 = input, the number of order statistics required; must
    be <= n/2.
ier = output, error indicator
    = 0 if no error detected
    = 1 if n <= 1.
    = 2 if n > 2000, in which case the order statistics
        are still calculated, but may be inaccurate.
    = 3 if n2 > n/2 (n.b. this differs from the
        published algorithm which returns an error
        if n2 is not equal to n/2.)

calls PDA_PPND16 = a variation of PPND7 in algorithm AS 241.

author: royston, j.p
PDA_PPND16
Returns the normal deviate corresponding to a given lower tail area of P

Invocation:

\[
\text{RESULT} = \text{PDA_PPND16}( \ P, \ IFAULT )
\]

Arguments:

\( P = \text{DOUBLE PRECISION} \) (Given)
Lower tail area (probability) of the normal distribution.

\( IFAULT = \text{INTEGER} \) (Returned)
Non-zero when cannot calculate result.

Returned Value:

\( \text{PDA_PPND16} = \text{DOUBLE PRECISION} \)
The required normal deviate.

Accuracy:
The result is accurate to about 1 part in \(10^{16}\).

Origin:
Applied Statistics / Statlib Archive
PDA_QSAx
Sort an array into ascending order

Description:
The routine uses the QUICKSORT algorithm to sort an array of values into ascending order. The “median of three” modification is included to reduce the likelihood of encountering the worst-case behaviour of QUICKSORT.
The routine exists for types REAL (x=R), DOUBLE PRECISION (x=D), and INTEGER (x=I).

Invocation:
   CALL PDA_QSAx( EL, X )

Arguments:
EL = INTEGER (Given)
The number of elements of X to sort.
X( EL ) = TYPE (Given and Returned)
The array to be sorted.

References:
Sedgwick, R., 1988, Algorithms (Addison-Wesley).

Timing:
If N elements are to be sorted, the average time goes as N*ln(N). The worst-case time goes as N**2.

Copyright:
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**PDA_QSDx**

Sort an array into descending order

**Description:**
The routine uses the QUICKSORT algorithm to sort an array of values into descending order. The “median of three” modification is included to reduce the likelihood of encountering the worst-case behaviour of QUICKSORT.

The routine exists for types REAL (x=R), DOUBLE PRECISION (x=D), and INTEGER (x=I).

**Invocation:**
```fortran
CALL PDA_QSDx( EL, X )
```

**Arguments:**
- **EL = INTEGER** (Given)
  The number of elements of X to be sorted.
- **X( EL ) = TYPE** (Given and Returned)
  The array to be sorted.

**References:**
- Sedgwick, R., 1988, Algorithms (Addison-Wesley).

**Timing:**
If N elements are to be sorted, the average time goes as N*ln(N). The worst-case time goes as N**2.

**Copyright:**
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PDA_QSIAx
Sort an array of pointers to access an array in ascending order

Description:
The routine uses the QUICKSORT algorithm to permute an array of pointers so that they access an associated array of values in ascending order. The “median of three” modification is included to reduce the likelihood of encountering the worst-case behaviour of QUICKSORT.

The routine exists for types REAL (x=R), DOUBLE PRECISION (x=D), and INTEGER (x=I).

Invocation:
CALL PDA_QSIAx( EL, X, IP )

Arguments:
EL = INTEGER (Given)
The number of elements of X to sort.
X( EL ) = TYPE (Given)
The array to be sorted.
IP( EL ) = INTEGER (Returned)
The indices of the elements of X in sorted order (i.e. IP( 1 ) gives the index into X of the lowest value).

References:
Sedgwick, R., 1988, Algorithms (Addison-Wesley).

Timing:
If N elements are to be sorted, the average time goes as N*ln(N). The worst-case time goes as N^2.

Copyright:
Copyright (C) 1992 Science & Engineering Research Council
PDA_QSIDx

Sort an array of pointers to access an array in descending order

Description:
The routine uses the QUICKSORT algorithm to permute an array of pointers so that they access an associated array of values in descending order. The “median of three” modification is included to reduce the likelihood of encountering the worst-case behaviour of QUICKSORT.

The routine exists for types REAL (x=R), DOUBLE PRECISION (x=D), and INTEGER (x=I).

Invocation:

CALL PDA_QSIDx( EL, X, IP )

Arguments:

EL = INTEGER (Given)
The number of pointers to be permuted.

X( EL ) = TYPE (Given)
The array to be sorted.

IP( EL ) = INTEGER (Given and Returned)
The indices of the elements of X in sorted order (i.e. IP( 1 ) gives the index into X of the highest value).

References:

Sedgwick, R., 1988, Algorithms (Addison-Wesley).

Timing:

If N elements are to be sorted, the average time goes as N*ln(N). The worst-case time goes as N**2.

Copyright:

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Convert FFTPACK Hermitian Fourier transform array into equivalent NAG array

Description:
This subroutine re-orders and normalises the supplied array of Fourier coefficients (as produced by FFTPACK subroutine PDA_RFFTF) so that the returned array looks like the equivalent array returned by NAG routine C06FAF.
The real and imaginary coefficients produced by PDA_RFFTF are numerically larger than the corresponding C06FAF coefficients by a factor of SQRT(NP), and are ordered differently. Both routines return A0 (the zeroth real term, i.e. the DC level in the array) in element 1. PDA_RFFTF then has corresponding real and imaginary terms in adjacent elements, whereas C06FAF has all the real terms together, followed by all the imaginary terms (in reverse order):

PDA_RFFTF: A0, A1, B1, A2, B2, A3, B3, ...
P06FAF: A0, A1, A2, A3, ..., ..., B3, B2, B1

The zeroth imaginary term (B0) always has the value zero and so is not stored in the array. Care has to be taken about the parity of the array size. If it is even, then there is one more real term than there is imaginary terms (excluding A0), i.e. if NP = 10, then the coefficients are stored as follows:

PDA_RFFTF: A0, A1, B1, A2, B2, A3, B3, A4, B4, A5
C06FAF: A0, A1, A2, A3, A4, A5, B4, B3, B2, B1

If NP = 9, then the coefficients are stored as follows:

PDA_RFFTF: A0, A1, B1, A2, B2, A3, B3, A4, B4
C06FAF: A0, A1, A2, A3, A4, B4, B3, B2, B1

Invocation:

CALL PDA_R2NAG( NP, R )

Arguments:

NP = INTEGER (Given)
The size of the array.

R( NP ) = REAL (Given and Returned)
The array holding the Fourier coefficients. Supplied in FFTPACK format and returned in NAG format.

Implementation Status:
A double precision version PDA_DR2NAG of the routine exists.
PDA_RAND

Returns pseudo-random numbers in the range 0 to 1

Description:
This is a simple random number generator providing deviates in the range 0 to 1, with period of
$2^{26}$, and to 6 or 7 digits accuracy. It is based upon Ahrens, Dieter & Grube’s TOMS599 routines.
Note that there is no STATUS argument for efficiency.

Invocation:
RESULT = PDA_RAND( X )

Arguments:
X = REAL (Given)
This is a dummy variable required by the Fortran standard.

Returned Value:
PDA_RAND = REAL
The pseudo-random deviate.

Prior Requirements:
The initial seed MUST be set using routine PDA_RNSED (equivalent to NAG’s G05CBF). If it has
not, there is no guarantee that sensible values will be returned from this function.

References:
PDA_RFFT

Backward transform of a real coefficient array.

Origin:

FFTPACK / NETLIB

Implementation Status:

A double precision version PDA_DRFFT of the routine has been added.
subroutine pda_rfftb(n,r,wsave)

subroutine pda_rfftb computes the real periodic sequence from its Fourier coefficients (Fourier synthesis). The transform is defined below at output parameter r.

Input parameters

n the length of the array r to be transformed. The method is most efficient when n is a product of small primes. n may change so long as different work arrays are provided.

r a real array of length n which contains the sequence to be transformed.

wsave a work array which must be dimensioned at least 2*n+15. In the program that calls pda_rfftb, the wsave array must be initialized by calling subroutine pda_rfft1(n,wsave) and a different wsave array must be used for each different value of n. This initialization does not have to be repeated so long as n remains unchanged thus subsequent transforms can be obtained faster than the first. The same wsave array can be used by pda_rfft1 and pda_rfftb.

Output parameters

r for n even and for i = 1,...,n

\[ r(i) = r(1) + (-1)^{i-1} r(n) \]

plus the sum from k=2 to k=n/2 of

\[ 2 \cdot r(2k-2) \cdot \cos((k-1)(i-1) \cdot 2\pi/n) \]
\[ -2 \cdot r(2k-1) \cdot \sin((k-1)(i-1) \cdot 2\pi/n) \]

for n odd and for i = 1,...,n

\[ r(i) = r(1) \text{ plus the sum from } k=2 \text{ to } k=(n+1)/2 \text{ of} \]

\[ 2 \cdot r(2k-2) \cdot \cos((k-1)(i-1) \cdot 2\pi/n) \]
\[ -2 \cdot r(2k-1) \cdot \sin((k-1)(i-1) \cdot 2\pi/n) \]

Note

This transform is unnormalized since a call of pda_rfft1 followed by a call of pda_rfftb will multiply the input sequence by n.
wsave contains results which must not be destroyed between calls of pda_rfftb or pda_rfftf.
PDA_RFFTF
Forward transform of a real periodic sequence.

Origin:
FFTPACK / NETLIB

Implementation Status:
A double precision version PDA_DRFFTF of the routine has been added.
**PDA_RFFTF**

subroutine pda_rfftf(n,r,wsave)

subroutine pda_rfftf computes the fourier coefficients of a real periodic sequence (fourier analysis). the transform is defined below at output parameter r.

**input parameters**

n the length of the array r to be transformed. the method is most efficient when n is a product of small primes. n may change so long as different work arrays are provided

r a real array of length n which contains the sequence to be transformed

wsave a work array which must be dimensioned at least 2*n+15. in the program that calls pda_rfftf. the wsave array must be initialized by calling subroutine pda_rfft(t)(n,wsave) and a different wsave array must be used for each different value of n. this initialization does not have to be repeated so long as n remains unchanged thus subsequent transforms can be obtained faster than the first. the same wsave array can be used by pda_rfftf and pda_rfft(t).

**output parameters**

r r(1) = the sum from i=1 to i=n of r(i)
   if n is even set l =n/2 , if n is odd set l = (n+1)/2
   then for k = 2,...,l
      r(2*k-2) = the sum from i = 1 to i = n of 
         r(i)*cos((k-1)*(i-1)*2*pi/n)
      r(2*k-1) = the sum from i = 1 to i = n of 
         -r(i)*sin((k-1)*(i-1)*2*pi/n)
   if n is even
      r(n) = the sum from i = 1 to i = n of 
         (-1)**(i-1)*r(i)

***** note
this transform is unnormalized since a call of pda_rfftf followed by a call of pda_rfft(t) will multiply the input
sequence by n.

wsave contains results which must not be destroyed between calls of pda_rfftf or pda_rfftb.
PDA_RFFTI
Initialize PDA_RFFTF and PDA_RFFTB.

Origin:
FFTPACK / NETLIB

Implementation Status:
A double precision version PDA_DRFFTI of the routine has been added.
subroutine pda_rffti(n, wsave)

subroutine pda_rffti initializes the array wsave which is used in both pda_rfft and pda_rfft_b. The prime factorization of n together with a tabulation of the trigonometric functions are computed and stored in wsave.

input parameter

n the length of the sequence to be transformed.

output parameter

wsave a work array which must be dimensioned at least 2*n+15.
the same work array can be used for both pda_rfft and pda_rfft_b as long as n remains unchanged. different wsave arrays are required for different values of n. the contents of wsave must not be changed between calls of pda_rfft or pda_rfft_b.
PDA_RINPx
Reorder an array in place using a permutation index

Description:
This routine reorders an array (in place) using an permutation vector. This is most likely the output from one of the sorting routines PDA_QSI[A|D] [x]

Invocation:
CALL PDA_RINP[x]( PERM, N, X, IFAIL )

Arguments:

PERM( N ) = INTEGER (Given and Returned)
The index vector. Note this is modified but should be returned in the same state as when input. Indices may not be negative.

N = INTEGER (Given)
Number of elements.

X( N ) = ? (Given and Returned)
The array to reorder.

IFAIL = INTEGER (Returned)
Status flag. Set 0 for success, otherwise the permutation isn’t correct.

Notes:

• Re-ordering is trivial if two arrays are available.

      DO I = 1, N
         XX( I ) = X( PERM( I )
      END DO
      The XX array contains the sorted values on completion.

• There is a routine for each of the data types integer, real and double precision; replace [x] in the routine name by I, R or D as appropriate. The data type of the X argument should match the routine being used.

Timing :
Proportional to N.
PDA_RNEXP

Returns pseudo-random numbers from an exponential distribution

Description:
This is a simple random-number generator providing deviates in the from an exponential distribution, with a period of $2^{26}$, and to 6 or 7 digits accuracy. It is based upon Ahrens, Dieter & Grube’s TOMS599 routines.

Invocation:
RESULT = PDA_RNEXP( X )

Arguments:
X = REAL (Given)
This is a dummy variable required by the Fortran standard.

Returned Value:
PDA_RNEXP = INTEGER
The pseudo-random deviate.

Prior Requirements:
The initial seed MUST be set using routine PDA_RNSED (equivalent to NAG’s G05CBF). If it has not, there is no guarantee that sensible values will be returned from this function.

References:
PDA_RNGAM

Returns pseudo-random numbers from a gamma distribution

Description:
This is a simple random-number generator providing deviates in the from a gamma distribution, with a period of $2^{26}$, and to 6 or 7 digits accuracy. It is based upon Ahrens, Dieter & Grube’s TOMS599 routines. A value of zero is returned if the argument of the gamma function is not positive.

Invocation:
RESULT = PDA_RNGAM( A )

Arguments:
A = REAL (Given)
The argument (mean) of the gamma function.

Returned Value:
PDA_RNGAM = REAL
The pseudo-random deviate. A value of zero is returned if the argument of the gamma function is not positive.

Prior Requirements:
The initial seed MUST be set using routine PDA_RNSED (equivalent to NAG’s G05CBF). If it has not, there is no guarantee that sensible values will be returned from this function.

References:
PDA_RNNOR

Returns pseudo-random numbers from a Gaussian distribution

Description:
This is a simple random-number generator providing deviates in the from a Gaussian distribution, with a period of $2^{26}$, and to 6 or 7 digits accuracy. It is based upon Ahrens, Dieter & Grube's TOMS599 routines.

Invocation:

\[
\text{RESULT} = \text{PDA_RNNOR}(\text{MEAN}, \text{SIGMA})
\]

Arguments:

MEAN = REAL (Given)
The mean value of the Gaussian distribution.

SIGMA = REAL (Given)
The standard deviation of the Gaussian distribution.

Returned Value:

PDA_RNNOR = REAL
The pseudo-random deviate.

Prior Requirements:
The initial seed MUST be set using routine PDA_RNSED (equivalent to NAG's G05CBF). If it has not, there is no guarantee that sensible values will be returned from this function.

References:
PDA_RNPOI
Returns pseudo-random numbers from a Poisson distribution

Description:
This is a simple random-number generator providing deviates in the from a Poisson distribution, with a period of 2**26, and to 6 or 7 digits accuracy. It is based upon Ahrens, Dieter & Grube’s TOMS599 routines.

Invocation:
RESULT = PDA_RNPOI( MEAN )

Arguments:
MEAN = REAL (Given)
The mean value of the Poisson distribution.

Returned Value:
PDA_RNPOI = INTEGER
The pseudo-random deviate. A value of -1 is returned if the supplied mean is not positive.

Prior Requirements:
The initial seed MUST be set using routine PDA_RNSED (equivalent to NAG’s G05CBF). If it has not, there is no guarantee that sensible values will be returned from this function.

References:
PDA_RNSED
Sets the seed for the PDA random-number generators

Description:
This sets the initial seed for the simple random-number generator based upon Ahrens, Dieter & Grube’s TOMS599 routines. The seed should be of the form 4*K+1, where K is a positive integer, and less than 2**28. When it is not, the nearest valid seed is used, but if this is negative, the seed becomes 2001.

Invocation:
CALL PDA_RNSED( SEED )

Arguments:
SEED = INTEGER (Given)
The random-number seed.

References:
PDA_SA
Continuous simulated annealing global optimisation algorithm.
Simple constraints can be specified.

Origin:
Module SIMANN from OPT / NETLIB

Implementation Status:
The routine now supports passing an external name for the objective function. It will also take a status argument set to zero and return it with value 1 if something goes wrong.
SUBROUTINE PDA_SA(FCN, 3 N,X,MAX,RT, EPS,NS,NT,NEPS,MAXEVL, LB, UB, C, IPRINT,
1    ISEED1,ISEED2,T,VM,XOPT,FOPT, NACC, NFCNEV, NOBDS, IER,
2    FSTAR, XP, NACP, STATUS)

Version: 3.2
Date: 1/22/94.
Differences compared to Version 2.0:
1. If a trial is out of bounds, a point is randomly selected
   from LB(i) to UB(i). Unlike in version 2.0, this trial is
   evaluated and is counted in acceptances and rejections.
   All corresponding documentation was changed as well.
Differences compared to Version 3.0:
1. If VM(i) > (UB(i) - LB(i)), VM is set to UB(i) - LB(i).
   The idea is that if T is high relative to LB & UB, most
   points will be accepted, causing VM to rise. But, in this
   situation, VM has little meaning; particularly if VM is
   larger than the acceptable region. Setting VM to this size
   still allows all parts of the allowable region to be selected.
Differences compared to Version 3.1:
1. Test made to see if the initial temperature is positive.
2. WRITE statements prettied up.
3. References to paper updated.
Minor update by Horst Meyerdierks, UoE, Starlink:
1. Make the function to be optimised an argument rather than using
   a constant name 'FCN'. This is the new first argument.

Synopsis:
This routine implements the continuous simulated annealing global
optimization algorithm described in Corana et al.'s article
"Minimizing Multimodal Functions of Continuous Variables with the
"Simulated Annealing" Algorithm" in the September 1987 (vol. 13,
no. 3, pp. 262-280) issue of the ACM Transactions on Mathematical
Software.

A very quick (perhaps too quick) overview of PDA_SA:
PDA_SA tries to find the global optimum of an N dimensional function.
It moves both up and downhill and as the optimization process
proceeds, it focuses on the most promising area.
To start, it randomly chooses a trial point within the step length
VM (a vector of length N) of the user selected starting point. The
function is evaluated at this trial point and its value is compared
to its value at the initial point.
In a maximization problem, all uphill moves are accepted and the
algorithm continues from that trial point. Downhill moves may be
accepted; the decision is made by the Metropolis criteria. It uses T
(temperature) and the size of the downhill move in a probabilistic
manner. The smaller T and the size of the downhill move are, the more
likely that move will be accepted. If the trial is accepted, the
algorithm moves on from that point. If it is rejected, another point
is chosen instead for a trial evaluation.
Each element of VM periodically adjusted so that half of all
function evaluations in that direction are accepted.
A fall in $T$ is imposed upon the system with the RT variable by $T(i+1) = RT \cdot T(i)$ where $i$ is the $i$th iteration. Thus, as $T$ declines, downhill moves are less likely to be accepted and the percentage of rejections rise. Given the scheme for the selection for VM, VM falls. Thus, as $T$ declines, VM falls and PDA_SA focuses upon the most promising area for optimization.

The importance of the parameter $T$:

The parameter $T$ is crucial in using PDA_SA successfully. It influences VM, the step length over which the algorithm searches for optima. For a small initial $T$, the step length may be too small; thus not enough of the function might be evaluated to find the global optima. The user should carefully examine VM in the intermediate output (set IPRINT = 1) to make sure that VM is appropriate. The relationship between the initial temperature and the resulting step length is function dependent.

To determine the starting temperature that is consistent with optimizing a function, it is worthwhile to run a trial run first. Set $RT = 1.5$ and $T = 1.0$. With $RT > 1.0$, the temperature increases and VM rises as well. Then select the $T$ that produces a large enough VM.

For modifications to the algorithm and many details on its use, (particularly for econometric applications) see Goffe, Ferrier and Rogers, "Global Optimization of Statistical Functions with Simulated Annealing," Journal of Econometrics, vol. 60, no. 1/2, Jan./Feb. 1994, pp. 65-100.

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As far as possible, the parameters here have the same name as in the description of the algorithm on pp. 266-8 of Corana et al.

In this description, SP is single precision, DP is double precision, INT is integer, L is logical and (N) denotes an array of length n. Thus, DP(N) denotes a double precision array of length n.

Input Parameters:

Note: The suggested values generally come from Corana et al. To drastically reduce runtime, see Goffe et al., pp. 90-1 for suggestions on choosing the appropriate RT and NT.

FCN - Function to be optimized. The form is

```
SUBROUTINE FCN(N,X,F)
INTEGER N
DOUBLE PRECISION X(N), F
...
function code with F = F(X)
...
RETURN
```
Note: This is the same form used in the multivariable minimization algorithms in the IMSL edition 10 library.

N - Number of variables in the function to be optimized. (INT)
X - The starting values for the variables of the function to be optimized. (DP(N))
MAX - Denotes whether the function should be maximized or minimized. A true value denotes maximization while a false value denotes minimization. Intermediate output (see IPRINT) takes this into account. (L)
RT - The temperature reduction factor. The value suggested by Corana et al. is .85. See Goffe et al. for more advice. (DP)
EPS - Error tolerance for termination. If the final function values from the last neps temperatures differ from the corresponding value at the current temperature by less than EPS and the final function value at the current temperature differs from the current optimal function value by less than EPS, execution terminates and IER = 0 is returned. (EP)
NS - Number of cycles. After NS*N function evaluations, each element of VM is adjusted so that approximately half of all function evaluations are accepted. The suggested value is 20. (INT)
NT - Number of iterations before temperature reduction. After NT*NS*N function evaluations, temperature (T) is changed by the factor RT. Value suggested by Corana et al. is MAX(100, 5*N). See Goffe et al. for further advice. (INT)
NEPS - Number of final function values used to decide upon termination. See EPS. Suggested value is 4. (INT)
MAXEVL - The maximum number of function evaluations. If it is exceeded, IER = 1. (INT)
LB - The lower bound for the allowable solution variables. (DP(N))
UB - The upper bound for the allowable solution variables. (DP(N))
If the algorithm chooses X(I) .LT. LB(I) or X(I) .GT. UB(I), I = 1, N, a point is from inside is randomly selected. This focuses the algorithm on the region inside UB and LB. Unless the user wishes to concentrate the search to a particular region, UB and LB should be set to very large positive and negative values, respectively. Note that the starting vector X should be inside this region. Also note that LB and UB are fixed in position, while VM is centered on the last accepted trial set of variables that optimizes the function.
C - Vector that controls the step length adjustment. The suggested value for all elements is 2.0. (DP(N))
IPRINT - controls printing inside PDA_SA. (INT)
Values: 0 - Nothing printed.
  1 - Function value for the starting value and summary results before each temperature reduction. This includes the optimal function value found so far, the total number of moves (broken up into uphill, downhill, accepted and rejected), the number of out of bounds trials, the number of new optima found at this temperature, the current optimal X and
the step length VM. Note that there are 
N*NS*WT function evaluations before each
temperature reduction. Finally, notice is
is also given upon achieving the termination
criteria.

2 - Each new step length (VM), the current optimal
X (XOPT) and the current trial X (X). This
gives the user some idea about how far X
strays from XOPT as well as how VM is adapting
to the function.

3 - Each function evaluation, its acceptance or
rejection and new optima. For many problems,
this option will likely require a small tree
if hard copy is used. This option is best
used to learn about the algorithm. A small
value for MAXEVL is thus recommended when
using IPRINT = 3.

Suggested value: 1
Note: For a given value of IPRINT, the lower valued
options (other than 0) are utilized.

ISEED1 - The first seed for the random number generator PDA_RANMAR.
0 .LE. ISEED1 .LE. 31328. (INT)

ISEED2 - The second seed for the random number generator PDA_RANMAR.
0 .LE. ISEED2 .LE. 30081. Different values for ISEED1
and ISEED2 will lead to an entirely different sequence
of trial points and decisions on downhill moves (when
maximizing). See Goffe et al. on how this can be used
to test the results of PDA_SA. (INT)

Input/Output Parameters:

T - On input, the initial temperature. See Goffe et al. for advice.
On output, the final temperature. (DP)

VM - The step length vector. On input it should encompass the
region of interest given the starting value X. For point
X(I), the next trial point is selected is from X(I) - VM(I)
to X(I) + VM(I). Since VM is adjusted so that about half
of all points are accepted, the input value is not very
important (i.e. is the value is off, PDA_SA adjusts VM to the
correct value). (DP(N))

STATUS - Should be given as zero. The value is unchanged, unless
an error occurs in PDA_RMARIN. In that case the return value
is one.

Output Parameters:

XOPT - The variables that optimize the function. (DP(N))
FOPT - The optimal value of the function. (DP)
NACC - The number of accepted function evaluations. (INT)
NFCNEV - The total number of function evaluations. In a minor
point, note that the first evaluation is not used in the
core of the algorithm; it simply initializes the
algorithm. (INT).

NOBDS - The total number of trial function evaluations that
would have been out of bounds of LB and UB. Note that
a trial point is randomly selected between LB and UB.
IER - The error return number. (INT)
Values:
0 - Normal return; termination criteria achieved.
1 - Number of function evaluations (NFCNEV) is greater than the maximum number (MAXEVL).
2 - The starting value (X) is not inside the bounds (LB and UB).
3 - The initial temperature is not positive.
99 - Should not be seen; only used internally.

Work arrays that must be dimensioned in the calling routine:
RWK1 (DP(NEPS)) (FSTAR in PDA_SA)
RWK2 (DP(N)) (XP " " )
IWK (INT(N)) (NACP " " )

Required Functions (included):
PDA_EXPRP - Replaces the function EXP to avoid under- and overflows. It may have to be modified for non IBM-type mainframes. (DP)
PDA_RMARIN - Initializes the random number generator PDA_RANMAR.
PDA_RANMAR - The actual random number generator. Note that PDA_RMARIN must run first (PDA_SA does this). It produces uniform random numbers on [0,1]. These routines are from Usenet's comp.lang.fortran. For a reference, see "Toward a Universal Random Number Generator" by George Marsaglia and Arif Zaman, Florida State University Report: FSU-SCRI-87-50 (1987). It was later modified by F. James and published in "A Review of Pseudo-random Number Generators." For further information, contact stuart@ads.com. These routines are designed to be portable on any machine with a 24-bit or more mantissa. I have found it produces identical results on a IBM 3081 and a Cray Y-MP.

Required Subroutines (included):
PDA_PRTVEC - Prints vectors.
PDA_PRT1 ... PDA_PRT10 - Prints intermediate output.

Machine Specific Features:
1. PDA_EXPRP may have to be modified if used on non-IBM type mainframes. Watch for under- and overflows in PDA_EXPRP.
2. Some FORMAT statements use G25.18; this may be excessive for some machines.
3. PDA_RMARIN and PDA_RANMAR are designed to be portable; they should not cause any problems.

Modification:
Use the new STATUS argument for the case that the seeds are out of range. (HME)
PDA_SAACx

Sorts the columns of a two dimensional array into ascending order

Description:
This routine returns a list of column sorted indices to an array (rows and columns span the first and second dimensions, respectively). This means that the data in the first column is sorted, any tied positions are then sorted by the corresponding values of the data in the second column, any tied values here are then sorted using the values in the third column and so on until the array is completely value ordered, or all columns have been used.

The sort is stable so any completely tied columns preserve their original order.

Invocation:
CALL PDA_SAAC[x]( A, NDEC, N, M, IP, LINK, IFAIL )

Arguments:
A( NDEC, M ) = ? (Given)
The matrix to be ranked column by column.

NDEC = INTEGER (Given)
The declared size of the first dimension of A.

N = INTEGER (Given)
The number of rows of A to be used.

M = INTEGER (Given)
The number of columns of A to be used. The declared size of this array should be at least two larger than this value (i.e. A should be at least A(NDEC,M+2)).

IP( M + 2 ) = INTEGER (Returned)
The indices of A when ranked into ascending order.

LINK( M + 2 ) = INTEGER (Given and Returned)
Workspace.

IFAIL = INTEGER (Returned)
Non zero if a bounds error has been detected.

Notes:

- There is a routine for each of the data types integer, real and double precision; replace [x] in the routine name by I, R or D as appropriate. The data type of the A argument should match the routine being used.
PDA_SAARx
Sorts the rows of a two dimensional array into ascending order

Description:
This routine returns a list of row sorted indices to an array (rows and columns span the first and second dimensions, respectively). This means that the data in the first row is sorted, any tied positions are then sorted by the corresponding values of the data in the second row, any tied values here are then sorted using the values in the third row and so on until the array is completely value ordered, or all rows have been used.

The sort is stable so any completely tied columns preserve their original order.

Invocation:
CALL PDA_SAAR[x]( A, NDEC, N, M, IP, LINK, IFAIL )

Arguments:
A( NDEC, M ) = ? (Given)
The matrix to be ranked row by row.

NDEC = INTEGER (Given)
The declared size of the first dimension of A. This should be two elements larger than the size of A to be sorted (i.e. N). The dimensions IP and LINK should also be declared as this size (i.e A should be at least A(N+2,M)).

N = INTEGER (Given)
The number of rows of A to be used (this should not be bigger than NDEC-2).

M = INTEGER (Given)
The number of columns of A to be used.

IP( NDEC ) = INTEGER (Returned)
The indices of A when ranked into ascending order.

LINK( NDEC ) = INTEGER (Given and Returned)
Workspace.

IFAIL = INTEGER (Returned)
Non zero if a bounds error has been detected.

Notes:
- There is a routine for each of the data types integer, real and double precision; replace [x] in the routine name by I, R or D as appropriate. The data type of the A argument should match the routine being used.
PDA_SPLDER
Evaluate derivative of spline, given in its B-spline representation from PDA_CURFIT.

Origin:
DIERCKX / NETLIB
subroutine pda_splder(t,n,c,k,nu,x,y,m,wrk,ier)

subroutine pda_splder evaluates in a number of points \( x(i) \), \( i=1,2,\ldots,m \) the derivative of order \( nu \) of a spline \( s(x) \) of degree \( k \), given in its b-spline representation.

calling sequence:
call pda_splder(t,n,c,k,nu,x,y,m,wrk,ier)

input parameters:
t : array, length \( n \), which contains the position of the knots.
n : integer, giving the total number of knots of \( s(x) \).
c : array, length \( n \), which contains the b-spline coefficients.
k : integer, giving the degree of \( s(x) \).
nu : integer, specifying the order of the derivative. \( 0 \leq nu \leq k \)
x : array, length \( m \), which contains the points where the derivative of \( s(x) \) must be evaluated.
m : integer, giving the number of points where the derivative of \( s(x) \) must be evaluated
wrk : real array of dimension \( n \). used as working space.

output parameters:
y : array, length \( m \), giving the value of the derivative of \( s(x) \) at the different points.
ier : error flag
   ier = 0 : normal return
   ier =10 : invalid input data (see restrictions)

restrictions:
\( 0 \leq nu \leq k \)
\( m \geq 1 \)
\( t(k+1) \leq x(i) \leq x(i+1) \leq t(n-k) \), \( i=1,2,\ldots,m-1 \).

other subroutines required: pda_fpbspl

references:
de boor c : on calculating with b-splines, j. approximation theory 6 (1972) 50-62.
cox m.g. : the numerical evaluation of b-splines, j. inst. maths applics 10 (1972) 134-149.

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PDA_SPLEV
Evaluate spline, given in its B-spline representation from PDA_CURFIT.

Origin:
DIERCKX / NETLIB
subroutine pda_splev(t,n,c,k,x,y,m,ier)

subroutine pda_splev evaluates in a number of points x(i), i=1,2,...,m a spline s(x) of degree k, given in its b-spline representation.

calling sequence:
   call pda_splev(t,n,c,k,x,y,m,ier)

input parameters:
   t : array,length n, which contains the position of the knots.
   n : integer, giving the total number of knots of s(x).
   c : array,length n, which contains the b-spline coefficients.
   k : integer, giving the degree of s(x).
   x : array,length m, which contains the points where s(x) must be evaluated.
   m : integer, giving the number of points where s(x) must be evaluated.

output parameter:
   y : array,length m, giving the value of s(x) at the different points.
   ier : error flag
      ier = 0 : normal return
      ier =10 : invalid input data (see restrictions)

restrictions:
   m >= 1
   t(k+1) <= x(i) <= x(i+1) <= t(n-k) , i=1,2,...,m-1.

other subroutines required: pda_fpbspl.

references :
   de boor c : on calculating with b-splines, j. approximation theory 6 (1972) 50-62.
   cox m.g. : the numerical evaluation of b-splines, j. inst. maths applics 10 (1972) 134-149.

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PDA_SPLINT
Calculate integral of spline, given its normalised B-spline representation from PDA_CURFIT.

Origin:
DIERCKX / NETLIB
real function pda_splint(t,n,c,k,a,b,wrk)

function pda_splint calculates the integral of a spline function \( s(x) \) of degree \( k \), which is given in its normalized b-spline representation

calling sequence:
\[
\text{aint} = \text{pda_splint}(t,n,c,k,a,b,\text{wrk})
\]

input parameters:
\[
\begin{align*}
t & : \text{array, length } n, \text{which contains the position of the knots of } s(x). \\
n & : \text{integer, giving the total number of knots of } s(x). \\
c & : \text{array, length } n, \text{ containing the b-spline coefficients.} \\
k & : \text{integer, giving the degree of } s(x). \\
a, b & : \text{real values, containing the end points of the integration interval. } s(x) \text{ is considered to be identically zero outside the interval } (t(k+1),t(n-k)).
\end{align*}
\]

output parameter:
\[
\begin{align*}
\text{aint} & : \text{real, containing the integral of } s(x) \text{ between } a \text{ and } b. \\
\text{wrk} & : \text{real array, length } n. \text{ used as working space on output, } \text{wrk} \text{ will contain the integrals of the normalized b-splines defined on the set of knots.}
\end{align*}
\]

other subroutines required: pda_fpintb.

references:
\[
\begin{align*}
gaffney \ p.w. & : \text{the calculation of indefinite integrals of b-splines} \\
& \text{j. inst. maths applics 17 (1976) 37-41.} \\
dierckx \ p. & : \text{curve and surface fitting with splines, monographs on numerical analysis, oxford university press, 1993.}
\end{align*}
\]

author:
\[
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\]

latest update : march 1987
PDA_SUBPLX
Subspace-searching simplex method for unconstrained optimization

Origin:
Module SUBPLEX from OPT / NETLIB

Description:
Subplex is a subspace-searching simplex method for the unconstrained optimization of general
multivariate functions. Like the Nelder-Mead simplex method it generalizes, the subplex method is
well suited for optimizing noisy objective functions. The number of function evaluations required
for convergence typically increases only linearly with the problem size, so for most applications
the subplex method is much more efficient than the simplex method.

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Reference:
Computer Sciences, University of Texas at Austin, 1990.
subroutine pda_subplx (f,n,tol,maxnfe,mode,scale,x,fx,nfe,
   *                   work,iwork,iflag)

Coded by Tom Rowan
Department of Computer Sciences
University of Texas at Austin

pda_subplx uses the subplex method to solve unconstrained optimization problems. The method is well suited for optimizing objective functions that are noisy or are discontinuous at the solution.

pda_subplx sets default optimization options by calling the subroutine pda_subopt. The user can override these defaults by calling pda_subopt prior to calling pda_subplx, changing the appropriate common variables, and setting the value of mode as indicated below.

By default, pda_subplx performs minimization.

input

f - user supplied function f(n,x) to be optimized,
   declared external in calling routine

n - problem dimension

tol - relative error tolerance for x (tol .ge. 0.)

maxnfe - maximum number of function evaluations

mode - integer mode switch with binary expansion
   (bit 1) (bit 0) :
   bit 0 = 0 : first call to pda_subplx
   = 1 : continuation of previous call
   bit 1 = 0 : use default options
   = 1 : user set options

scale - scale and initial step sizes for corresponding components of x
   (If scale(1) .lt. 0.,
   abs(scale(1)) is used for all components of x,
   and scale(2),...,scale(n) are not referenced.)

x - starting guess for optimum

work - double precision work array of dimension .ge.
   2*n + nsmax*(nsmax+4) + 1
   (nsmax is set in subroutine pda_subopt.
   default: nsmax = min(5,n))

iwork - integer work array of dimension .ge.
\[ n + \text{int}(n/\text{nsmin}) \]
(nsmin is set in subroutine pda_subopt.
default: \text{nsmin} = \text{min}(2,n))

**output**

- **x** - computed optimum
- **fx** - value of \( f \) at \( x \)
- **nfe** - number of function evaluations
- **iflag** - error flag
  - \(-2\) : invalid input
  - \(-1\) : maxnfe exceeded
  - \(0\) : tol satisfied
  - \(1\) : limit of machine precision
  - \(2\) : fstop reached (fstop usage is determined
    by values of options minf, nfstop, and
    irepl. default: \( f(x) \) not tested against
    fstop)

iflag should not be reset between calls to pda_subplx.
PDA_SUMSL
Unconstrained minimisation of a smooth non-linear function of n variables, function and gradients supplied.

Origin:
Module SUMSL (algorithm 611) from TOMS

Author:
David M. Gay,

Reference:
subroutine pda_sumsl(n, d, x, calcf, calcg, iv, liv, lv, v, 
1                  uiparm, urparm, ufparm)

Minimize general unconstrained objective function using analytic
gradient and hessian approx. from secant update.

-------------------------- parameter usage --------------------------

n........ (input) the number of variables on which f depends, i.e.,
the number of components in x.

d........ (input/output) a scale vector such that d(i)*x(i),
i = 1,2,...,n, are all in comparable units.
d can strongly affect the behavior of pda_sumsl.
finding the best choice of d is generally a trial-
and-error process. choosing d so that d(i)*x(i)
has about the same value for all i often works well.
the defaults provided by subroutine pda_deflt (see iv
below) require the caller to supply d.

x........ (input/output) before (initially) calling pda_sumsl, the call-
er should set x to an initial guess at x*. when
pda_sumsl returns, x contains the best point so far
found, i.e., the one that gives the least value so
far seen for f(x).

calcf.... (input) a subroutine that, given x, computes f(x). calcf
must be declared external in the calling program.
it is invoked by
               call calcf(n, x, nf, f, uiparm, urparm, ufparm)
when calcf is called, nf is the invocation
count for calcf. nf is included for possible use
with calcg. if x is out of bounds (e.g., if it
would cause overflow in computing f(x)), then calcf
should set nf to 0. this will cause a shorter step
to be attempted. (if x is in bounds, then calcf
should not change nf.) the other parameters are as
described above and below. calcf should not change
n, p, or x.

calcg.... (input) a subroutine that, given x, computes g(x), the gra-
dient of f at x. calcg must be declared external in
the calling program. it is invoked by
               call calcg(n, x, nf, g, uiparm, urparm, ufparm)
when calcg is called, nf is the invocation
count for calcf at the time f(x) was evaluated. the
x passed to calcg is usually the one passed to calcf
on either its most recent invocation or the one
prior to it. if calcf saves intermediate results
for use by calcg, then it is possible to tell from
nf whether they are valid for the current x (or
which copy is valid if two copies are kept). if g
cannot be computed at x, then calcg should set nf to
0. in this case, pda_sumsl will return with iv(1) = 65.
(if g can be computed at x, then calcg should not
changed nf.) the other parameters to calcg are as
described above and below. calcg should not change
iv........ (input/output) an integer value array of length liv (see
below) that helps control the pda_sumsl algorithm and
that is used to store various intermediate quanti-
ties. of particular interest are the initialization/
return code iv(1) and the entries in iv that control
printing and limit the number of iterations and func-
tion evaluations. see the section on iv input
values below.

liv...... (input) length of iv array. must be at least 60. if liv
is too small, then pda_sumsl returns with iv(1) = 15.
when pda_sumsl returns, the smallest allowed value of
liv is stored in iv(lastiv) -- see the section on
iv output values below. (this is intended for use
with extensions of pda_sumsl that handle constraints.)

lv....... (input) length of v array. must be at least 71+n*(n+15)/2.
(at least 77+n*(n+17)/2 for pda_smsno, at least
78+n*(n+12) for pda_humsl). if lv is too small, then
pda_sumsl returns with iv(1) = 16. when pda_sumsl returns,
the smallest allowed value of lv is stored in
iv(lastv) -- see the section on iv output values
below.

v........ (input/output) a floating-point value array of length lv
(see below) that helps control the pda_sumsl algorithm
and that is used to store various intermediate
quantities. of particular interest are the entries
in v that limit the length of the first step
attempted (lmax0) and specify convergence tolerances
(afctol, lmaxs, rfctol, sctol, xctol, xftol).

uiparm... (input) user integer parameter array passed without change
to calcf and calcg.

urparm... (input) user floating-point parameter array passed without
change to calcf and calcg.

ufparm... (input) user external subroutine or function passed without
change to calcf and calcg.

*** iv input values (from subroutine pda_deflt) ***

iv(1)... on input, iv(1) should have a value between 0 and 14......
0 and 12 mean this is a fresh start. 0 means that
pda_deflt(2, iv, liv, lv, v)
is to be called to provide all default values to iv and
v. 12 (the value that pda_deflt assigns to iv(1)) means the
caller has already called pda_deflt and has possibly changed
some iv and/or v entries to non-default values.
13 means pda_deflt has been called and that pda_sumsl (and
pda_sumit) should only do their storage allocation. that is,
they should set the output components of iv that tell
where various subarrays arrays of v begin, such as iv(g)
(and, for pda_humsl and pda_humit only, iv(dtol)), and return.
14 means that a storage has been allocated (by a call
with iv(1) = 13) and that the algorithm should be
started. when called with iv(1) = 13, pda_sumsl returns
iv(1) = 14 unless liv or lv is too small (or n is not
positive). default = 12.

iv(inith).... iv(25) tells whether the hessian approximation h should be initialized. 1 (the default) means pda_sumit should initialize h to the diagonal matrix whose i-th diagonal element is d(i)**2. 0 means the caller has supplied a cholesky factor l of the initial hessian approximation h = l*(l**t) in v, starting at v(iv(lmat)) = v(iv(42)) (and stored compactly by rows). note that iv(lmat) may be initialized by calling pda_sumsl with iv(1) = 13 (see the iv(1) discussion above). default = 1.

iv(mxcal)... iv(17) gives the maximum number of function evaluations (calls on calcf) allowed. if this number does not suffice, then pda_sumsl returns with iv(1) = 9. default = 200.

iv(mxiter)... iv(18) gives the maximum number of iterations allowed. it also indirectly limits the number of gradient evaluations (calls on calcg) to iv(mxiter) + 1. if iv(mxiter) iterations do not suffice, then pda_sumsl returns with iv(1) = 10. default = 150.

iv(outlev)... iv(19) controls the number and length of iteration summary lines printed (by pda_itsum). iv(outlev) = 0 means do not print any summary lines. otherwise, print a summary line after each abs(iv(outlev)) iterations. if iv(outlev) is positive, then summary lines of length 78 (plus carriage control) are printed, including the following... the iteration and function evaluation counts, f = the current function value, relative difference in function values achieved by the latest step (i.e., reldf = (f0-v(f))/f01, where f01 is the maximum of abs(v(f)) and abs(v(f0)) and v(f0) is the function value from the previous iteration), the relative function reduction predicted for the step just taken (i.e., preldf = v(preduc) / f01, where v(preduc) is described below), the scaled relative change in x (see v(reldx) below), the step parameter for the step just taken (stppar = 0 means a full newton step, between 0 and 1 means a relaxed newton step, between 1 and 2 means a double dogleg step, greater than 2 means a scaled down cauchy step -- see subroutine dbldog), the 2-norm of the scale vector d times the step just taken (see v(dstnrm) below), and npreldf, i.e., v(nreduc)/f01, where v(nreduc) is described below -- if npreldf is positive, then it is the relative function reduction predicted for a newton step (one with stppar = 0). if npreldf is negative, then it is the negative of the relative function reduction predicted for a step computed with step bound v(lmaxs) for use in testing for singular convergence.

if iv(outlev) is negative, then lines of length 50 are printed, including only the first 6 items listed above (through reldx).

default = 1.

iv(parprt)... iv(20) = 1 means print any nondefault v values on a fresh start or any changed v values on a restart. iv(parprt) = 0 means skip this printing. default = 1.

iv(prunit)... iv(21) is the output unit number on which all printing
is done.  \textit{iv(prunit)} = 0 (the default) means suppress all printing.

\textit{iv(solprt)} = 1 means print out the value of \(x\) returned (as well as the gradient and the scale vector \(d\)).
\textit{iv(solprt)} = 0 means skip this printing.  default = 1.

\textit{iv(statpr)} = 1 means print summary statistics upon returning.  these consist of the function value, the scaled relative change in \(x\) caused by the most recent step (see \textit{v(reldx)} below), the number of function and gradient evaluations (calls on \textit{calcf} and \textit{calcg}), and the relative function reductions predicted for the last step taken and for a newton step (or perhaps a step bounded by \textit{v(lmaxs)}) -- see the descriptions of \textit{preldf} and \textit{npreldf} under \textit{iv(outlev)} above).
\textit{iv(statpr)} = 0 means skip this printing.
\textit{iv(statpr)} = -1 means skip this printing as well as that of the one-line termination reason message.  default = 1.

\textit{iv(x0prt)} = 1 means print the initial \(x\) and scale vector \(d\) (on a fresh start only).  \textit{iv(x0prt)} = 0 means skip this printing.  default = 1.

\*\*\* (selected) \textit{iv} output values  \*\*\*

\textit{iv(1)} is a return code....

3 = \textit{x}-convergence.  the scaled relative difference (see \textit{v(reldx)}) between the current parameter vector \(x\) and a locally optimal parameter vector is very likely at most \textit{v(xctol)}.

4 = relative function convergence.  the relative difference between the current function value and its locally optimal value is very likely at most \textit{v(rfctol)}.

5 = both \textit{x}- and relative function convergence (i.e., the conditions for \textit{iv(1)} = 3 and \textit{iv(1)} = 4 both hold).

6 = absolute function convergence.  the current function value is at most \textit{v(afctol)} in absolute value.

7 = singular convergence.  the hessian near the current iterate appears to be singular or nearly so, and a step of length at most \textit{v(lmaxs)} is unlikely to yield a relative function decrease of more than \textit{v(sctol)}.

8 = false convergence.  the iterates appear to be converging to a noncritical point.  this may mean that the convergence tolerances (\textit{v(afctol)}, \textit{v(rfctol)}, \textit{v(xctol)}) are too small for the accuracy to which the function and gradient are being computed, that there is an error in computing the gradient, or that the function or gradient is discontinuous near \(x\).

9 = function evaluation limit reached without other convergence (see \textit{iv(mxfcal)}).

10 = iteration limit reached without other convergence (see \textit{iv(mxiter)}).

11 = \textit{pda_stopx} returned .\texttt{true.}  (external interrupt).  see the usage notes below.

14 = storage has been allocated (after a call with \textit{iv(1)} = 13).
17 = restart attempted with n changed.
18 = d has a negative component and iv(dtype) le. 0.
19...43 = v(iv(1)) is out of range.
63 = f(x) cannot be computed at the initial x.
64 = bad parameters passed to assess (which should not occur).
65 = the gradient could not be computed at x (see calcg above).
67 = bad first parameter to pda_deflt.
80 = iv(1) was out of range.
81 = n is not positive.
iv(g)........ iv(28) is the starting subscript in v of the current gradient vector (the one corresponding to x).
iv(lastiv)... iv(44) is the least acceptable value of liv. (it is only set if liv is at least 44.)
iv(lastv).... iv(45) is the least acceptable value of lv. (it is only set if lv is large enough, at least iv(lastiv)).
iv(nfcall)... iv(6) is the number of calls so far made on calcf (i.e., function evaluations).
iv(ngcall)... iv(30) is the number of gradient evaluations (calls on calcg).
iv(niter).... iv(31) is the number of iterations performed.

*** (selected) v input values (from subroutine pda_deflt) ***

v(bias)..... v(43) is the bias parameter used in subroutine dbldog -- see that subroutine for details. default = 0.8.
v(afctol)... v(31) is the absolute function convergence tolerance. if pda_sumsl finds a point where the function value is less than v(afctol) in absolute value, and if pda_sumsl does not return with iv(1) = 3, 4, or 5, then it returns with iv(1) = 6. this test can be turned off by setting v(afctol) to zero. default = max(10**-20, machep**2), where machep is the unit roundoff.
v(dinit).... v(38), if nonnegative, is the value to which the scale vector d is initialized. default = -1.
v(lmax0).... v(35) gives the maximum 2-norm allowed for d times the very first step that pda_sumsl attempts. this parameter can markedly affect the performance of pda_sumsl.
v(lmaxs)..... v(36) is used in testing for singular convergence -- if the function reduction predicted for a step of length bounded by v(lmaxs) is at most v(sctol) * abs(f0), where f0 is the function value at the start of the current iteration, and if pda_sumsl does not return with iv(1) = 3, 4, 5, or 6, then it returns with iv(1) = 7. default = 1.
v(rfctol)... v(32) is the relative function convergence tolerance. if the current model predicts a maximum possible function reduction (see v(nreduc)) of at most v(rfctol)*abs(f0) at the start of the current iteration, where f0 is the then current function value, and if the last step attempted achieved no more than twice the predicted function decrease, then pda_sumsl returns with iv(1) = 4 (or 5). default = max(10**-10, machep**(2/3)), where machep is the unit roundoff.
v(sctol).... v(37) is the singular convergence tolerance -- see the description of v(lmaxs) above.
v(tuner1).... v(26) helps decide when to check for false convergence. This is done if the actual function decrease from the current step is no more than v(tuner1) times its predicted value. default = 0.1.
v(xctol).... v(33) is the x-convergence tolerance. if a newton step (see v(nreduc)) is tried that has v(reldx) .le. v(xctol) and if this step yields at most twice the predicted function decrease, then pda_sumsl returns with iv(1) = 3 (or 5). (see the description of v(reldx) below.) default = machep**0.5, where machep is the unit roundoff.
v(xftol).... v(34) is the false convergence tolerance. if a step is tried that gives no more than v(tuner1) times the predicted function decrease and that has v(reldx) .le. v(xftol), and if pda_sumsl does not return with iv(1) = 3, 4, 5, 6, or 7, then it returns with iv(1) = 8. (see the description of v(reldx) below.) default = 100*machep, where machep is the unit roundoff.
v(*)........ pda_deflt supplies to v a number of tuning constants, with which it should ordinarily be unnecessary to tinker. see section 17 of version 2.2 of the nl2sol usage summary (i.e., the appendix to ref. 1) for details on v(i), i = decfac, incfac, phmnmfc, phmxfc, rdfcmn, rdfcmx, tuner2, tuner3, tuner4, tuner5.

*** (selected) v output values ***
v(dgnorm)... v(1) is the 2-norm of (diag(d)**-1)*g, where g is the most recently computed gradient.
v(dstnrm)... v(2) is the 2-norm of diag(d)*step, where step is the current step.
v(f)........ v(10) is the current function value.
v(f0)....... v(13) is the function value at the start of the current iteration.
v(nreduc)... v(6), if positive, is the maximum function reduction possible according to the current model, i.e., the function reduction predicted for a newton step (i.e., step = -h**-1 * g, where g is the current gradient and h is the current hessian approximation).
   if v(nreduc) is negative, then it is the negative of the function reduction predicted for a step computed with a step bound of v(lmaxs) for use in testing for singular convergence.
v(preduc)... v(7) is the function reduction predicted (by the current quadratic model) for the current step. this (divided by v(f0)) is used in testing for relative function convergence.
v(reldx).... v(17) is the scaled relative change in x caused by the current step, computed as
   max(abs(d(i)*(x(i)-x0(i)), 1 .le. i .le. p) /
   max(d(i)*abs(x(i))+abs(x0(i))), 1 .le. i .le. p),
where x = x0 + step.
*** algorithm notes ***

this routine uses a hessian approximation computed from the bfgs update (see ref 3). only a cholesky factor of the hessian approximation is stored, and this is updated using ideas from ref. 4. steps are computed by the double dogleg scheme described in ref. 2. the steps are assessed as in ref. 1.

*** usage notes ***

after a return with iv(1) .le. 11, it is possible to restart, i.e., to change some of the iv and v input values described above and continue the algorithm from the point where it was interrupt-ed. iv(1) should not be changed, nor should any entries of iv and v other than the input values (those supplied by pda_deflt).

those who do not wish to write a calcg which computes the gradient analytically should call pda_smsno rather than pda_sumsl. pda_smsno uses finite differences to compute an approximate gradient.

those who would prefer to provide f and g (the function and gradient) by reverse communication rather than by writing subrou-tines calcf and calcg may call on pda_sumit directly. see the com-ments at the beginning of pda_sumit.

those who use pda_sumsl interactively may wish to supply their own pda_stopx function, which should return .true. if the break key has been pressed since pda_stopx was last invoked. this makes it possible to externally interrupt pda_sumsl (which will return with iv(1) = 11 if pda_stopx returns .true.).

storage for g is allocated at the end of v. thus the caller may make v longer than specified above and may allow calcg to use elements of g beyond the first n as scratch storage.

*** portability notes ***

the pda_sumsl distribution tape contains both single- and double-precision versions of the pda_sumsl source code, so it should be un-necessary to change precisions.

only the functions pda_imdcon and pda_rmdcon contain machine-dependent constants. to change from one machine to another, it should suffice to change the (few) relevant lines in these functions.

intrinsic functions are explicitly declared. on certain com-puters (e.g. univac), it may be necessary to comment out these declarations. so that this may be done automatically by a simple program, such declarations are preceded by a comment having c/ in columns 1-3 and blanks in columns 4-72 and are followed by a comment having c/ in columns 1 and 2 and blanks in columns 3-72.

the pda_sumsl source code is expressed in 1966 ansi standard fortran. it may be converted to fortran 77 by commenting out all lines that fall between a line having c/6 in columns 1-3 and a line having c/7 in columns 1-3 and by removing (i.e., replacing by a blank) the c in column 1 of the lines that follow the c/7 line and precede a line having c/ in columns 1-2 and blanks in columns 3-72. these changes convert some data statements into
parameter statements, convert some variables from real to character*4, and make the data statements that initialize these variables use character strings delimited by primes instead of hollerith constants. (Such variables and data statements appear only in modules pda_itsum and pda_parck. Parameter statements appear nearly everywhere.) These changes also add save statements for variables given machine-dependent constants by pda_rmdcon.

*** references ***

1. dennis, j.e., gay, d.m., and welsch, r.e. (1981), algorithm 573 -- an adaptive nonlinear least-squares algorithm, acm trans. math. software 7, pp. 369-383.

2. dennis, j.e., and mei, h.h.w. (1979), two new unconstrained optimization algorithms which use function and gradient values, j. optim. theory applic. 28, pp. 453-482.


*** general ***

coded by david m. gay (winter 1980). revised summer 1982. this subroutine was written in connection with research supported in part by the national science foundation under grants mcs-7600324, dcr75-10143, 76-14311dss, mcs76-11989, and mcs-7906671.
PDA_SURFIT
Find a bivariate spline approximation to irregularly spaced 2-D data.

Origin:
DIERCKX / NETLIB
Description:
Given the set of data points \((x(i), y(i), z(i))\) and the set of positive numbers \(w(i), i=1,...,m\), subroutine pda_surfit determines a smooth bivariate spline approximation \(s(x,y)\) of degrees \(k_x\) and \(k_y\) on the rectangle \(x_b \leq x \leq x_e, y_b \leq y \leq y_e\).

If \(iopt = -1\) pda_surfit calculates the weighted least-squares spline according to a given set of knots.

If \(iopt \geq 0\) the total numbers \(n_x\) and \(n_y\) of these knots and their position \(t_x(j), j=1,...,n_x\) and \(t_y(j), j=1,...,n_y\) are chosen automatically by the routine. The smoothness of \(s(x,y)\) is then achieved by minimizing the discontinuity jumps in the derivatives of \(s(x,y)\) across the boundaries of the sub-panels \((t_x(i), t_x(i+1)) \times (t_y(j), t_y(j+1))\). The amount of smoothness is determined by the condition that \(f(p) = \sum ((w(i) \cdot (z(i) - s(x(i), y(i))))^2) \leq s\), with \(s\) a given non-negative constant, called the smoothing factor.

The fit is given in the b-spline representation (b-spline coefficients \(c((n_y-ky-1) \cdot (i-1)+j), i=1,...,nx-kx-1; j=1,...,ny-ky-1\)) and can be evaluated by means of subroutine bispev.

Calling Sequence:
```
call pda_surfit( iopt, m, x, y, z, w, xb, xe, yb, ye, kx, ky, s, nxest, nyest, nmax, eps, nx, tx, ny, ty, c, fp, wrk1, lwrk1, wrk2, lwrk2, iwrk, kwrk, ier)
```

Parameters:
- \(iopt\) : Integer flag. On entry \(iopt\) must specify whether a weighted least-squares spline (\(iopt=-1\)) or a smoothing spline (\(iopt=0\) or 1) must be determined. If \(iopt=0\) the routine will start with an initial set of knots:

\[
\begin{align*}
tx(i) &= xb, tx(i+kx+1) = xe, i=1,...,kx+1; \\
ty(i) &= yb, ty(i+ky+1) = ye, i=1,...,ky+1.
\end{align*}
\]

If \(iopt=1\) the routine will continue with the set of knots found at the last call of the routine. Attention: a call with \(iopt=1\) must always be immediately preceded by another call with \(iopt=1\) or \(iopt=0\). Unchanged on exit.

- \(m\) : Integer. On entry \(m\) must specify the number of data points. \(m \geq (k_x+1) \cdot (k_y+1)\). Unchanged on exit.

- \(x\) : Real array of dimension at least \((m)\).

- \(y\) : Real array of dimension at least \((m)\).

- \(z\) : Real array of dimension at least \((m)\).

Before entry, \(x(i), y(i), z(i)\) must be set to the co-ordinates of the \(i\)-th data point, for \(i=1,...,m\). The order of the data points is immaterial. Unchanged on exit.

- \(w\) : Real array of dimension at least \((m)\). Before entry, \(w(i)\) must be set to the \(i\)-th value in the set of weights. The \(w(i)\) must be strictly positive. Unchanged on exit.
\( \text{x}, \text{xe} \): Real values. On entry \( \text{xb}, \text{xe}, \text{yb} \), \( \text{ye} \) must specify the boundaries of the rectangular approximation domain. \( \text{xb}_\leq \text{x}(i) \leq \text{xe}, \text{yb}_\leq \text{y}(i) \leq \text{ye}, i=1, \ldots, m \). Unchanged on exit.

\( \text{kb}, \text{ky} \): Integer values. On entry \( \text{kb} \) and \( \text{ky} \) must specify the degree of the spline. \( 1 \leq \text{kb}, \text{ky} \leq 5 \). It is recommended to use bi-cubic \( (\text{kb} = \text{ky} = 3) \) splines. Unchanged on exit.

\( s \): Real. On entry (in case \( \text{iopt} \geq 0 \)) \( s \) must specify the smoothing factor. \( s > 0 \). Unchanged on exit. For advice on the choice of \( s \) see further comments.

\( \text{nxest} \) : Integer. Unchanged on exit.
\( \text{nyest} \) : Integer. Unchanged on exit.

On entry, \( \text{nxest} \) and \( \text{nyest} \) must specify an upper bound for the number of knots required in the \( x \)- and \( y \)-directions respect. These numbers will also determine the storage space needed by the routine. \( \text{nxest} \geq 2*(\text{kb}+1) \), \( \text{nyest} \geq 2*(\text{ky}+1) \). In most practical situation \( \text{nxest} = \text{kb}+1+\sqrt{m/2} \), \( \text{nyest} = \text{ky}+1+\sqrt{m/2} \) will be sufficient. See also further comments.

\( \text{nmax} \) : Integer. On entry \( \text{nmax} \) must specify the actual dimension of the arrays \( \text{tx} \) and \( \text{ty} \). \( \text{nmax} > = \text{nxest} \), \( \text{nmax} > = \text{nyest} \). Unchanged on exit.

\( \text{eps} \): Real. On entry, \( \text{eps} \) must specify a threshold for determining the effective rank of an over-determined linear system of equations. \( 0 < \text{eps} < 1 \). If the number of decimal digits in the computer representation of a real number is \( q \), then \( 10^{-q} \) is a suitable value for \( \text{eps} \) in most practical applications. Unchanged on exit.

\( \text{nx} \) : Integer. Unless \( \text{ier}=10 \) (in case \( \text{iopt} \geq 0 \)), \( \text{nx} \) will contain the total number of knots with respect to the \( x \)-variable, of the spline approximation returned. If the computation mode \( \text{iopt}=1 \) is used, the value of \( \text{nx} \) should be left unchanged between subsequent calls. In case \( \text{iopt}=-1 \), the value of \( \text{nx} \) should be specified on entry.

\( \text{tx} \): Real array of dimension \( \text{nmax} \). On successful exit, this array will contain the knots of the spline with respect to the \( x \)-variable, i.e. The position of the interior knots \( \text{tx}(\text{kb}+2), \ldots, \text{tx}(\text{nx}-\text{kb}-1) \) as well as the position of the additional knots \( \text{tx}(1)=\ldots=\text{tx}(\text{kb}+1)=\text{xb} \) and \( \text{tx}(\text{nx}-\text{kb})=\ldots=\text{tx}(\text{nx})=\text{xe} \) needed for the b-spline representation. If the computation mode \( \text{iopt}=1 \) is used, the values of \( \text{tx}(1), \ldots, \text{tx}(\text{nx}) \) should be left unchanged between subsequent calls. If the computation mode \( \text{iopt}=-1 \) is used, the values \( \text{tx}(\text{kb}+2), \ldots, \text{tx}(\text{nx}-\text{kb}-1) \) must be supplied by the user, before entry. See also the restrictions (\( \text{ier}=10 \)).

\( \text{ny} \) : Integer. Unless \( \text{ier}=10 \) (in case \( \text{iopt} \geq 0 \)), \( \text{ny} \) will contain the total number of knots with respect to the \( y \)-variable, of the
PDA_SURFIT

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spline approximation returned. If the computation mode iopt = 1 is used, the value of ny should be left unchanged between subsequent calls. In case iopt = -1, the value of ny should be specified on entry.

\textbf{ty} : Real array of dimension nmax. On successful exit, this array will contain the knots of the spline with respect to the y-variable, i.e. The position of the interior knots \( ty(ky+2), ..., ty(ny-ky-1) \) as well as the position of the additional knots \( ty(1) = ..., ty(ky+1) = ye \) and \( ty(ny-ky) = ..., ty(ny) = ye \) needed for the b-spline representation. If the computation mode iopt = 1 is used, the values of \( ty(1), ..., ty(ny) \) should be left unchanged between subsequent calls. If the computation mode iopt = -1 is used, the values \( ty(ky+2), ..., ty(ny-ky-1) \) must be supplied by the user, before entry. See also the restrictions (ier = 10).

\textbf{c} : Real array of dimension at least \((nxest-kx-1)*(nyest-ky-1)\). On successful exit, c contains the coefficients of the spline approximation \( s(x,y) \). 

\textbf{fp} : Real. Unless ier = 10, fp contains the weighted sum of squared residuals of the spline approximation returned.

\textbf{wrk1} : Real array of dimension (lwrk1). Used as workspace. If the computation mode iopt = 1 is used the value of wrk1(1) should be left unchanged between subsequent calls. On exit \( \text{wrk1}(2), \text{wrk1}(3), ..., \text{wrk1}(1+(nx-kx-1)*(ny-ky-1)) \) will contain the values \( d(i)/\max(d(i)), i=1, ..., (nx-kx-1)*(ny-ky-1) \) with \( d(i) \) the \( i \)-th diagonal element of the reduced triangular matrix for calculating the b-spline coefficients. It includes those elements whose square is less than eps, which are treated as 0 in the case of presumed rank deficiency (ier < -2).

\textbf{lwrk1} : Integer. On entry lwrk1 must specify the actual dimension of the array wrk1 as declared in the calling (sub)program. lwrk1 must not be too small. Let:

\[
\begin{align*}
    u &= \text{nxest}-kx-1, \quad v = \text{nyest}-ky-1, \quad km = \max(kx,ky)+1, \\
    ne &= \max(\text{nxest},\text{nyest}), \quad bx = kx*v+ky+1, \quad by = ky*u+kx+1, \\
    \text{if}(bx.le.by) \quad b1 &= bx, \quad b2 = b1+v-kx \\
    \text{if}(bx.gt.by) \quad b1 &= by, \quad b2 = b1+u-kx
\end{align*}
\]

then

\[
\text{lwrk1} \geq u*v*(2+b1+b2)+2*(u+v+km*(m+ne)+ne-kx-ky)+b2+1
\]

\textbf{wrk2} : Real array of dimension (lwrk2). Used as workspace, but only in the case a rank deficient system is encountered.

\textbf{lwrk2} : Integer. On entry lwrk2 must specify the actual dimension of the array wrk2 as declared in the calling (sub)program. lwrk2 > 0 . A safe upper bound for \( \text{lwrk2} = u*v*(b2+1)+b2 \) where \( u, v \) and \( b2 \) are as above. If there are enough data points, scattered uniformly over the approximation domain
and if the smoothing factor s is not too small, there is a
good chance that this extra workspace is not needed. A lot
of memory might therefore be saved by setting lwrk2=1.
(see also ier > 10).

iwrk : integer array of dimension (kwrk). Used as workspace.

kwrk : Integer. On entry kwrk must specify the actual dimension of
the array iwrk as declared in the calling (sub)program.
kwrk >= m+(nxest-2*kx-1)*(nyest-2*ky-1).

ier : Integer. Unless the routine detects an error, ier contains a
non-positive value on exit:
0 : Normal return. The spline returned has a residual sum of
squares fp such that abs(fp-s)/s <= tol with tol a
relative tolerance set to 0.001 by the program.

-1 : Normal return. The spline returned is an interpolating
spline (fp=0).

-2 : Normal return. The spline returned is the weighted least
squares polynomial of degrees kx and ky. In this extreme
case fp gives the upper bound for the smoothing factor s.

< -2 : Warning. The coefficients of the spline returned have been
computed as the minimal norm least-squares solution of a
(numerically) rank deficient system. (-ier) gives the rank.
Especially if the rank deficiency which can be computed as
(nx-kx-1)*(ny-ky-1)+ier, is large the results may be
inaccurate. They could also seriously depend on the value of
eps.

1 : Error. The required storage space exceeds the available
storage space, as specified by the parameters nxest and
nyest. Probable causes - nxest or nyest too small. If these
parameters are already large, it may also indicate that s is
too small. The approximation returned is the weighted
least-squares spline according to the current set of knots.
The parameter fp gives the corresponding weighted sum of
squared residuals (fp>s).

2 : Error. A theoretically impossible result was found during
the iteration process for finding a smoothing spline with
fp = s. Probable causes - s too small or badly chosen eps.
There is an approximation returned but the corresponding
weighted sum of squared residuals does not satisfy the
condition abs(fp-s)/s < tol.

3 : Error. The maximal number of iterations maxit (set to 20
by the program) allowed for finding a smoothing spline
with fp=s has been reached. Probable causes - s too small
there is an approximation returned but the corresponding
weighted sum of squared residuals does not satisfy the
condition $\text{abs}(fp-s)/s < \text{tol}$.

4 : Error. No more knots can be added because the number of b-spline coefficients $(nx-kx-1)\times(ny-ky-1)$ already exceeds the number of data points $m$. Probable causes - either $s$ or $m$ too small. The approximation returned is the weighted least-squares spline according to the current set of knots. The parameter $fp$ gives the corresponding weighted sum of squared residuals ($fp>s$).

5 : Error. No more knots can be added because the additional knot would (quasi) coincide with an old one. Probable causes - $s$ too small or too large a weight to an inaccurate data point. The approximation returned is the weighted least-squares spline according to the current set of knots. The parameter $fp$ gives the corresponding weighted sum of squared residuals ($fp>s$).

10 : Error. On entry, the input data are controlled on validity. The following restrictions must be satisfied:

\begin{align*}
-1 & \leq \text{iopt} \leq 1, \quad 1 \leq kx,ky \leq 5, \quad m = (kx+1)\times(ky+1), \quad nxest > 2\times kx+2, \\
nyest & > 2\times ky+2, \quad 0 < \text{eps} < 1, \quad nmax = \text{nxest}, \quad \text{nmax} = \text{nyest}, \\
\text{xb} & > x(i) < \text{xe}, \quad \text{yb} < y(i) < \text{ye}, \quad w(i) > 0, \quad i=1,...,m \\
lwrk1 & > u\times v\times(2\times b1+b2)+2\times(u+v+km\times(m+ne)+ne-kx-ky)+b2+1 \\
kwrk & > m\times(nxest-2\times kx-1)\times(nyest-2\times ky-1)
\end{align*}

if $\text{iopt} = -1$: $2\times kx+2 \leq nx = \text{nxest}$
\begin{align*}
\text{xb} & < x(kx+2) < x(kx+3) < \ldots < x(nx-kx-kx) < x(kx+2) < \text{ny} = \text{nyest} \\
\text{yb} & < y(ky+2) < y(ky+3) < \ldots < y(ny-ky-kx) < y(ky+2)
\end{align*}

if $\text{iopt} = 0$: $s > 0$

If one of these conditions is found to be violated, control is immediately repassed to the calling program. In that case there is no approximation returned.

> 10 : Error. $lwrk2$ is too small, i.e. There is not enough work space for computing the minimal least-squares solution of a rank deficient system of linear equations. $\text{ier}$ gives the requested value for $lwrk2$. There is no approximation returned but, having saved the information contained in $nx$, $ny,tx,ty,wrk1$, and having adjusted the value of $lwrk2$ and the dimension of the array $wrk2$ accordingly, the user can continue at the point the program was left, by calling $\text{pda\_surfit}$ with $\text{iopt}=1$.

Further Comments:

By means of the parameter $s$, the user can control the tradeoff between closeness of fit and smoothness of fit of the approximation. If $s$ is too large, the spline will be too smooth and signal will be lost; if $s$ is too small the spline will pick up too much noise. In the extreme cases the program will return an interpolating spline if $s=0$ and the weighted least-squares polynomial (degrees $kx,ky$) if $s$ is very large.
Between these extremes, a properly chosen \( s \) will result in a good compromise between closeness of fit and smoothness of fit. To decide whether an approximation, corresponding to a certain \( s \) is satisfactory the user is highly recommended to inspect the fits graphically.

Recommended values for \( s \) depend on the weights \( w(i) \). If these are taken as \( 1/d(i) \) with \( d(i) \) an estimate of the standard deviation of \( z(i) \), a good \( s \)-value should be found in the range \((m-\sqrt{2m},m+\sqrt{2m})\). If nothing is known about the statistical error in \( z(i) \) each \( w(i) \) can be set equal to one and \( s \) determined by trial and error, taking account of the comments above. The best is then to start with a very large value of \( s \) (to determine the least-squares polynomial and the corresponding upper bound \( fp0 \) for \( s \)) and then to progressively decrease the value of \( s \) (say by a factor 10 in the beginning, i.e. \( s=fp0/10, fp0/100,... \) and more carefully as the approximation shows more detail) to obtain closer fits. To choose \( s \) very small is strongly discouraged. This considerably increases computation time and memory requirements. It may also cause rank-deficiency (ier<-2) and endanger numerical stability.

To economize the search for a good \( s \)-value the program provides different modes of computation. At the first call of the routine, or whenever he wants to restart with the initial set of knots the user must set iopt=0.

If iopt=1 the program will continue with the set of knots found at the last call of the routine. This will save a lot of computation time if pda_surfit is called repeatedly for different values of \( s \). The number of knots of the spline returned and their location will depend on the value of \( s \) and on the complexity of the shape of the function underlying the data. If the computation mode iopt=1 is used, the knots returned may also depend on the \( s \)-values at previous calls (if these were smaller). Therefore, if after a number of trials with different \( s \)-values and iopt=1, the user can finally accept a fit as satisfactory, it may be worthwhile for him to call pda_surfit once more with the selected value for \( s \) but now with iopt=0. Indeed, pda_surfit may then return an approximation of the same quality of fit but with fewer knots and therefore better if data reduction is also an important objective for the user.

The number of knots may also depend on the upper bounds nxest and nyest. Indeed, if at a certain stage in pda_surfit the number of knots in one direction (say nx) has reached the value of its upper bound (nxest), then from that moment on all subsequent knots are added in the other (y) direction. This may indicate that the value of nxest is too small. On the other hand, it gives the user the option of limiting the number of knots the routine locates in any direction for example, by setting nxest=\( 2*kx+2 \) (the lowest allowable value for nxest), the user can indicate that he wants an approximation which is a simple polynomial of degree \( kx \) in the variable x.

Other Subroutines Required:
- pda_fpback, pda_fpbspl, pda_fpsurf, pda_fpdisc, pda_fpgivs, pda_fprank,
- pda_fprati, pda_fprota, pda_fporde
References:
Dierckx P. : "An algorithm for surface fitting with spline functions"
Dierckx P. : "An algorithm for surface fitting with spline functions"
Dierckx P. : "Curve and surface fitting with splines", monographs on
numerical analysis, Oxford University Press, 1993.

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Creation Date : may 1979
latest update : march 1987
PDA_UNCMND

Minimise smooth non-linear function of n variables, function values only.

Origin:
NMS / TIBER

Implementation Status:
The warning messages are no longer printed. The same information is returned in the argument INFO.
SUBROUTINE PDA_UNCMND (N,X0,FCN,X,F,INFO,W,LW)

***BEGIN PROLOGUE PDA_UNCMND
***DATE WRITTEN 870923 (YYMMDD)
***REVISION DATE 871222 (YYMMDD)
***CATEGORY NO. G1B1A1
***KEYWORDS UNCONSTRAINED MINIMIZATION
***AUTHOR NASH, S.G., (GEORGE MASON UNIVERSITY)
***PURPOSE PDA_UNCMND minimizes a smooth nonlinear function of n variables.
   A subroutine that computes the function value at any point
   must be supplied, but derivative values are not required.
   PDA_UNCMND provides a simple interface to more flexible lower
   level routines. User has no control over options.

***DESCRIPTION
   From the book, "Numerical Methods and Software" by
   D. Kahaner, C. Moler, S. Nash
   Prentice Hall, 1988

   This routine uses a quasi-Newton algorithm with line search
   to minimize the function represented by the subroutine FCN.
   At each iteration, the nonlinear function is approximated
   by a quadratic function derived from a Taylor series.
   The quadratic function is minimized to obtain a search direction,
   and an approximate minimum of the nonlinear function along
   the search direction is found using a line search. The
   algorithm computes an approximation to the second derivative
   matrix of the nonlinear function using quasi-Newton techniques.

   The PDA_UNCMND package is quite general, and provides many options
   for the user. However, this subroutine is designed to be
   easy to use, with few choices allowed. For example:

   1. Only function values need be computed. First derivative
      values are obtained by finite-differencing. This can be
      very costly when the number of variables is large.

   2. It is assumed that the function values can be obtained
      accurately (to an accuracy comparable to the precision of
      the computer arithmetic).

   3. At most 150 iterations are allowed.

   4. It is assumed that the function values are well-scaled,
      that is, that the optimal function value is not pathologically
      large or small.

   For more information, see the reference listed below.

PARAMETERS
----------
N   --> INTEGER
    Dimension of problem
X0(N) --> DOUBLE PRECISION
   Initial estimate of minimum

FCN --> Name of routine to evaluate minimization function.
   Must be declared EXTERNAL in calling routine, and
   have calling sequence
   SUBROUTINE FCN(N, X, F)
   with N and X as here, F the computed function value.

X(N) <-- DOUBLE PRECISION
   Local minimum

F <-- DOUBLE PRECISION
   Function value at local minimum X

INFO <-- INTEGER
   Termination code
   INFO = 0: Optimal solution found
   INFO = 1: Terminated with gradient small,
            X is probably optimal
   INFO = 2: Terminated with step size small,
            X is probably optimal
   INFO = 3: Lower point cannot be found,
            X is probably optimal
   INFO = 4: Iteration limit (150) exceeded
   INFO = 5: Too many large steps,
            function may be unbounded
   INFO = -1: Insufficient workspace

W(LW) --> DOUBLE PRECISION
   Workspace

LW --> INTEGER
   Size of workspace, at least N*(N+10)

***REFERENCES  R.B. SCHNABEL, J.E. KOONTZ, AND BE.E. WEISS, A MODULAR
   SYSTEM OF ALGORITHMS FOR UNCONSTRAINED MINIMIZATION,
   REPORT CU-CS-240-82, COMP. SCI. DEPT., UNIV. OF
   COLORADO AT BOULDER, 1982.

***MODIFICATION
   950404 Remove calls to XERROR, which are only level-0 messages that
   duplicate information in the INFO argument. (HME).

***ROUTINES CALLED PDA_OPTDRD, XERROR

***END PROLOGUE  PDA_UNCMND
PDA_V11
Calculates an approximation to the variance of the largest normal order statistic

Description:
Calculates an approximation to the variance of the largest normal order statistic for a sample of a given size. This routine is used to estimate the value of the V11 argument of PDA_COVMAT.

Invocation:
RESULT = PDA_V11( N, IFAULT )

Arguments:
N = INTEGER (Given)
The size of the order sample.

IFault = INTEGER (Returned)
Zero for success, otherwise N is less than 1.

Returned Value:
PDA_V11 = DOUBLE PRECISION
The required variance.

Origin:
Applied Statistics / Statlib Archive

Copyright:
The Royal Statistical Society.
Description:
PDA_XERMSG processes a diagnostic message in a manner determined by the value of LEVEL. In the original, things also depended on an error report control flag KONTRL. This was by default 2. If KONTRL was zero or negative, no information other than the message itself (including numeric values, if any) would have been printed. If KONTRL was positive, introductory messages, tracebacks, etc., would have been printed in addition to the message. Depending on KONTRL and LEVEL the error handling mechanism might also have included aborting the program via a STOP statement.

<table>
<thead>
<tr>
<th>ABS(KONTRL)</th>
<th>LEVEL</th>
<th>0</th>
<th>1</th>
<th>2</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>fatal</td>
<td>fatal</td>
<td>fatal</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>not printed</td>
<td>printed</td>
<td>fatal</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>not printed</td>
<td>printed</td>
<td>printed</td>
<td></td>
</tr>
<tr>
<td>-1</td>
<td>not printed</td>
<td>printed</td>
<td>printed</td>
<td></td>
</tr>
</tbody>
</table>

In the current version, this routine will always issue a message via EMS_REP. Under no circumstances is the program aborted. Instead this routine always returns control to the caller after setting the STATUS argument (which is new in this version) to 1.

Invocation:

CALL PDA_XERMSG( LIBRAR, SUBROU, MESSG, NERR, LEVEL, STATUS )

Arguments:

LIBRAR = CHARACTER * ( * ) (Given)
The name of the library such as ‘SLATEC’. This will form part of the message put out.

SUBROU = CHARACTER * ( * ) (Given)
The name of the subroutine calling PDA_XERMSG. This will form part of the message put out.

MESSG = CHARACTER * ( * ) (Given)
The principal error or warning message.

NERR = INTEGER (Given)
Ignored.

LEVEL = INTEGER (Given)
Ignored.
STATUS = INTEGER (Returned)
The global status. Always returned as 1.

References:
R. E. Jones and D. K. Kahaner, XERROR, the SLATEC Error-handling Package, SAND82-0800, Sandia Laboratories, 1982.

Implementation Status:
The newline sentinel $$ is not interpreted by this routine.
B Adding further routines

This section describes how a new routine or set of routines might be added to the existing library. The development of the library is coordinated by David Berry.

1. Investigate the functionality in NAG in a particular area as currently used by applications. Consult the NAG documentation for this.

2. Consider what functionality is actually necessary. This is often an abstraction from what NAG provides: Say, NAG may have a choice of routines or algorithms, or a set of NAG routines may be closely related.

3. Look at what is available in the Public Domain. Use the GAMS decision tree on the World Wide Web and give preference to SLATEC routines. Also give preference to double precision routines.

4. Make up your mind as to which routines from the Public Domain will be necessary and how they will cover completely and consistently a particular area of functionality. You will now have a list of one or a few user-callable routines that you want to incorporate into the library.

5. Down-load the code for each user-callable routine you picked. Make sure you get all code needed, including subsidiary routines. Use the ‘fullsource’ anchor in GAMS. Check that the transfer was successful, Mosaic will not warn you if the final five per cent are missing.

6. Go through the single-file source code and change the routine names. Whenever possible the old names should just be preceded by ‘PDA_’.

7. Split the full source into Fortran modules with ‘fsplit’.

8. If the routine PDA_XERMSG is part of the full source, SLATEC error handling is involved. If PDA_XERROR is present the old SLATEC error handling is involved. Remove PDA_XERMSG or PDA_XERROR and their subsidiaries PDA_FDUMP, PDA_J4SAVE, PDA_XERABT, PDA_XERCNT, PDA_XERCTL, PDA_XERHTL, PDA_XERPRN, PDA_XERPRT, PDA_XERRWV, PDA_XERSAV, PDA_XERSVE, PDA_XGETUA. Also remove PDA_I1MACH.

9. Check if there are routine name conflicts with the existing library. Check if modules of the same name are compatible or identical. Remove the redundant routines.

10. Modify the error handling and message output.

   • If PDA_XERMSG is used to report and handle errors, an inherited status must be introduced to the routines in question. The calls to PDA_XERMSG must be given an extra integer argument. This status must be passed down and up all the way from the user application to PDA_XERMSG. The user application is assumed to give a value of zero indicating OK. When PDA_XERMSG is called it changes the status to one indicating an error. You must also check that the new routines you want to introduce into the library obey the status. When a routine A calls a routine B which has the status as argument, then the status may have to be checked by A. If the status is bad, A must return gracefully to its caller. In the original SLATEC library XERMSG may be called with error severity levels that cause XERMSG to stop the program. This does not and must not happen in this library. The code you down-loaded may rely on the abortion to have occurred, but the code that goes into this library must not rely on this.

   • If PDA_XERROR is called, these calls should be re-directed to PDA_XERMSG or avoided altogether. If using PDA_XERMSG, review the routines accordingly.

   • The routines in this library are not allowed to execute STOP statements or to write messages to the ‘terminal’. They must instead return a status code indicating what went wrong or what message the caller may or may not want to pass on to the user.
You can introduce new calls to PDA_XERMSG to issue error reports, but this is not encouraged.

(11) Register the new source files in the ‘makefile’. Also register any test programs. Test programs havecapitalised names, library routines have lower-case names.

(12) Modify pda_test.f to make a trivial call to the user-callable routines that you introduced. pda_test.f can be compiled and linked to see if all modules necessary are present, it cannot be executed.

(13) Update the source of this document.

- Did you tap into a Public Domain package so far unused in the library?
- Were there problems with routine names?
- Do the new routines use include files?
- Provide migration hints, if possible.
- Add the user-callable routines to the list of routines and provide the routine documentation.

If you write new code for the library:

- Try not to use include files.
- Try not to use common blocks.
- Do not execute STOP statements.
- Do not write to the ‘terminal’.
- Do not call routines outside this library, do not call EMS_REP, ERR_REP, MSG_OUT, etc.
- Handle errors by returning a status to the caller that indicates what went wrong or what message might have to be delivered to the user. Do not deliver messages from code for this library. If you have to, use PDA_XERMSG.
- Adhere to the Starlink Application Programming Standard (SGP/16).

C  Changes

C.1  Changes from version 0.4 to 0.5

A new routine PDA_SUMSL has been added which performs unconstrained minimisation allowing gradients to be supplied in addition to function values.

C.2  Changes from version 0.3 to 0.4

- The use of the higher level ERR and MSG error reporting libraries within PDA has been replaced by use of the lower level EMS library (see section 7).
- The following areas have had new routines added:
  - Matrices.
  - Minimisation.
  - Normal distribution.
  - Pseudo-random numbers.
  - Simple statistics.
– Sorting.
– Two-dimensional interpolation and fitting.

The new routines are:

- **PDA_BISPEV** - Evaluates the bivariate spline approximation found by PDA_SURFIT.
- **PDA_CHE2D** - Evaluates a 2-dimensional Chebyshev polynomial.
- **PDA_COVMAT** - Approximates the covariance matrix of normal order statistics.
- **PDA_DB2INK** - Determines a piecewise polynomial function that interpolates 2-D gridded data.
- **PDA_DB2VAL** - Evaluates the function found by PDA_DB2INK.
- **PDA_DCOV** - Calculates the covariance matrix for a nonlinear data fitting problem.
- **PDA_DNLS1** - Minimises the sum of squares of M non-linear functions.
- **PDA_DNLS1E** - Minimises the sum of squares of M non-linear functions (easy version).
- **PDA_DQED** - Solves bounded nonlinear least squares and nonlinear equations.
- **PDA_IDBVIP** - Performs 2-D bivariate interpolation on irregularly scattered data.
- **PDA_IDSFFT** - Performs smooth surface fitting on irregular data.
- **PDA_IPERM** - Forms the inverse of a permutation.
- **PDA_LSQR** - Solves sparse unsymmetric linear, and damped, least squares problems.
- **PDA_NSCOR** - Calculates the approximate expected values of normal order statistics.
- **PDA_PPND16** - Returns the normal deviate corresponding to a given lower tail area of P.
- **PDA_QSAx** - Sort an array into ascending order.
- **PDA_QSDx** - Sort an array into descending order.
- **PDA_QSIAx** - Sort an array of pointers to access an array in ascending order.
- **PDA_QSIDx** - Sort an array of pointers to access an array in descending order.
- **PDA_RAND** - Returns pseudo-random numbers in the range 0 to 1.
- **PDA_RNEXP** - Returns pseudo-random numbers from an exponential distribution.
- **PDA_RNGAM** - Returns pseudo-random numbers from a gamma distribution.
- **PDA_RNNOR** - Returns pseudo-random numbers from a Gaussian distribution.
- **PDA_RNPOI** - Returns pseudo-random numbers from a Poisson distribution.
- **PDA_RNSED** - Sets the seed for the PDA random-number generators.
- **PDA_RINP** - Reorders an array in place using a permutation index.
- **PDA_SAAC** - Sorts the columns of a two dimensional array into ascending order.
- **PDA_SAAR** - Sorts the rows of a two dimensional array into ascending order.
- **PDA_SURFIT** - Find a bivariate spline approximation to irregularly spaced 2-D data.
- **PDA_V11** - Calculates an approximation to the variance of the largest normal order statistic.